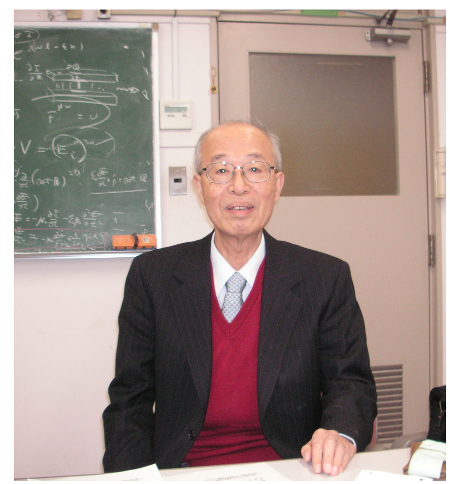
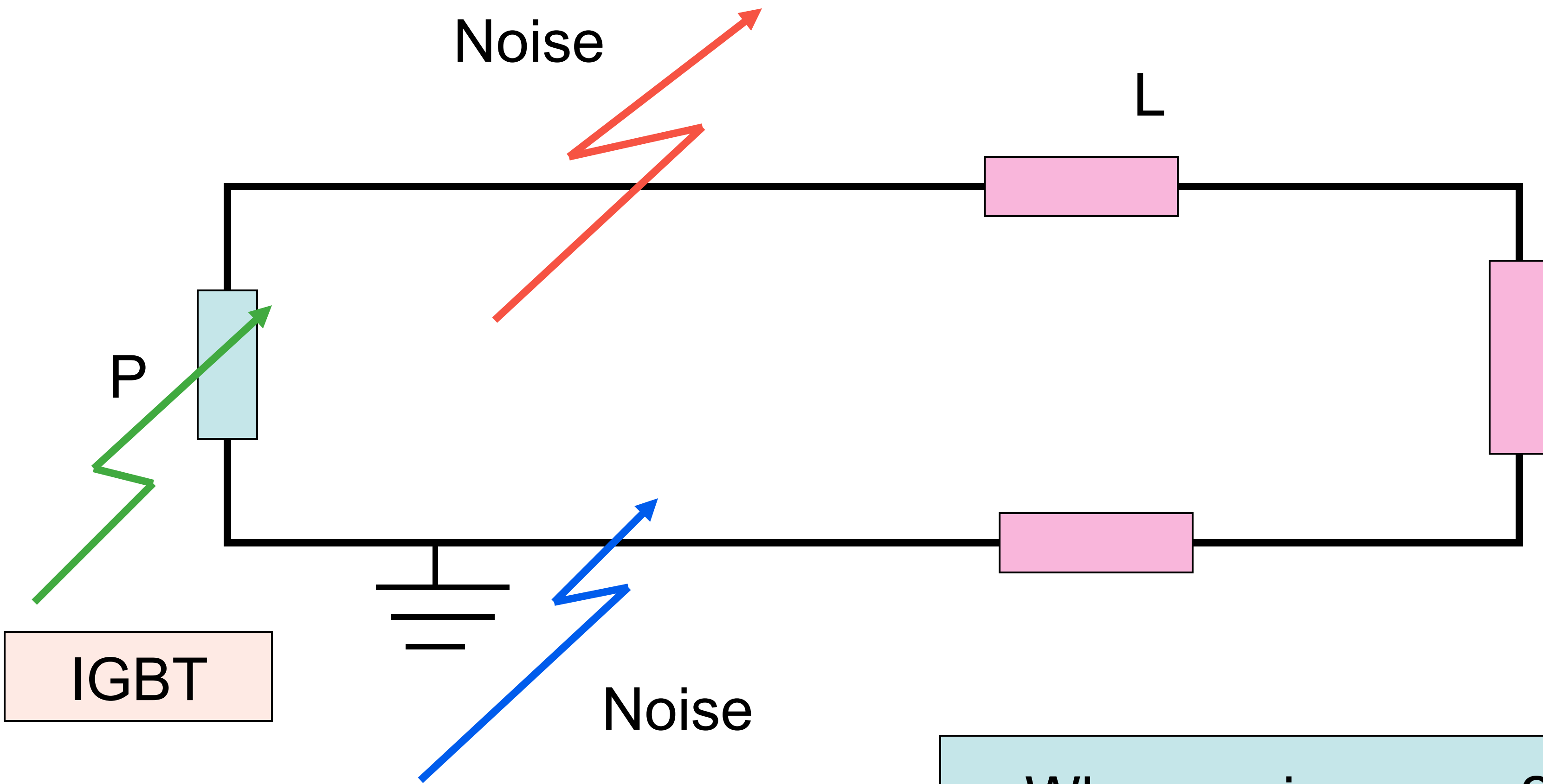
New Multiconductor Transmission-line Theory and The Mechanism of Noise Reduction

Hiroshi Toki
Research Center for Nuclear Physics (RCNP) Osaka University
High Energy Accelerator Research Organization (KEK)

(Collaborator)
Kenji Sato
RCNP, NIRS, KEK



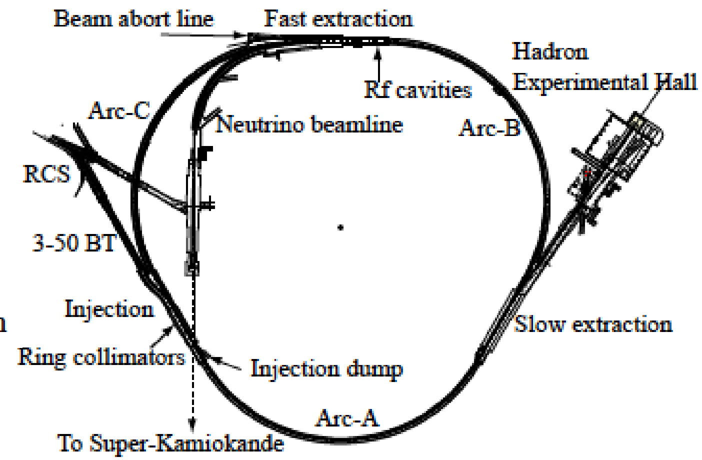
Standard electric circuit



Where noise come?

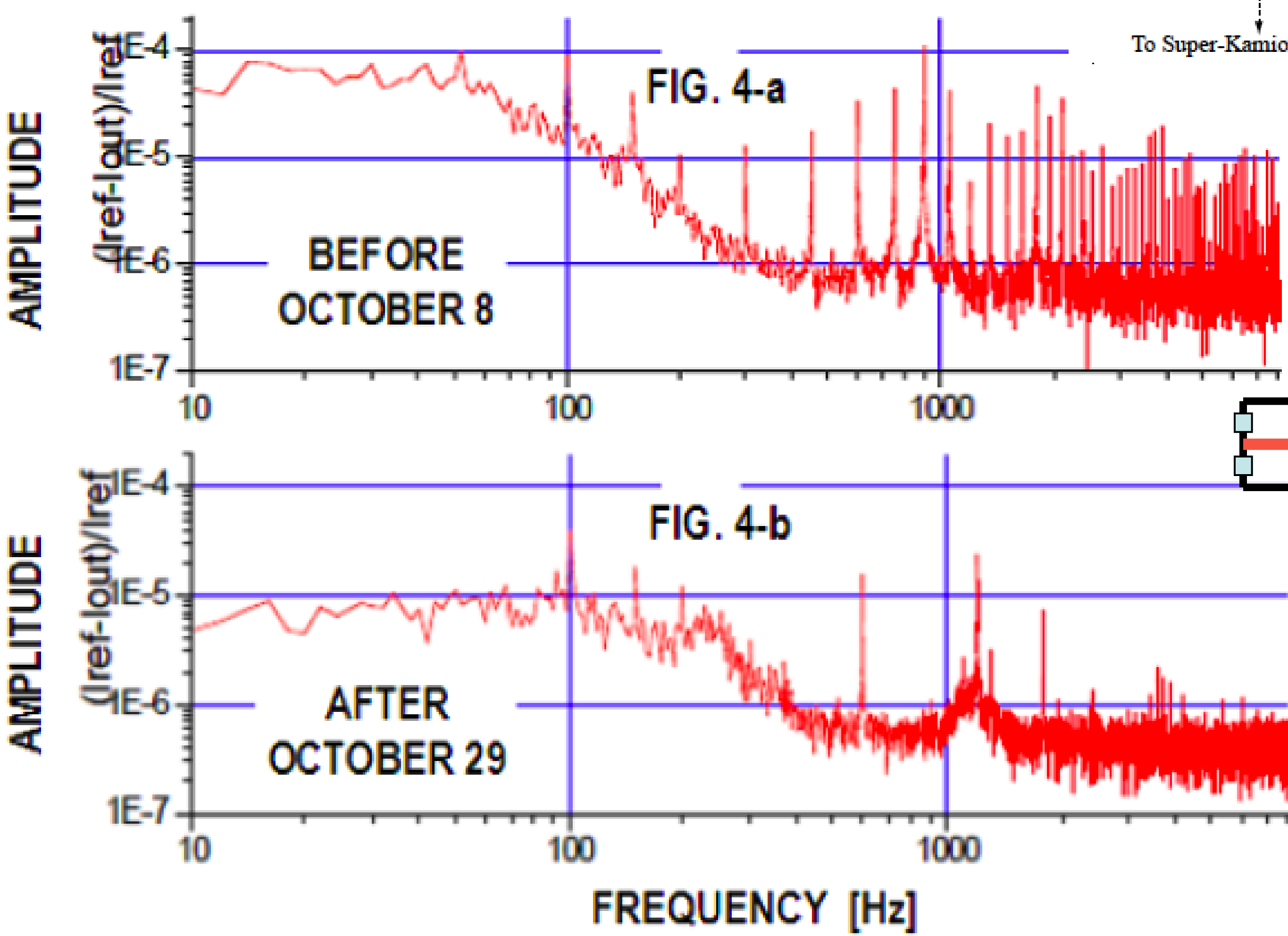
大強度陽子加速器(J−PARC : KEK)

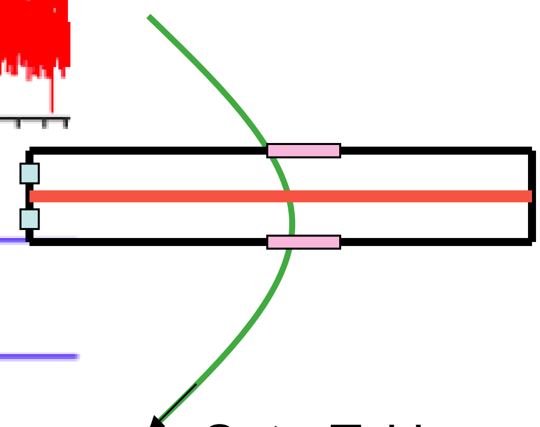
Noise vs. frequency of J-PARC-MR



BEAM COMMISSIONING OF THE J-PARC MAIN RING

H. Kobayashi for the J-PARC MR group, J-PARC Center KEK and JAEA, Tokai, Ibaraki, Japan

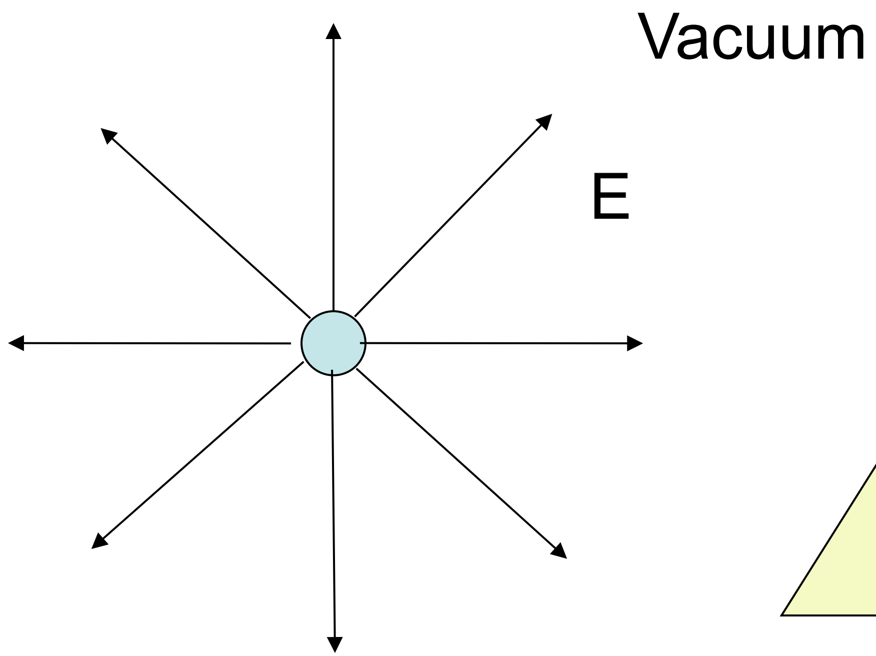


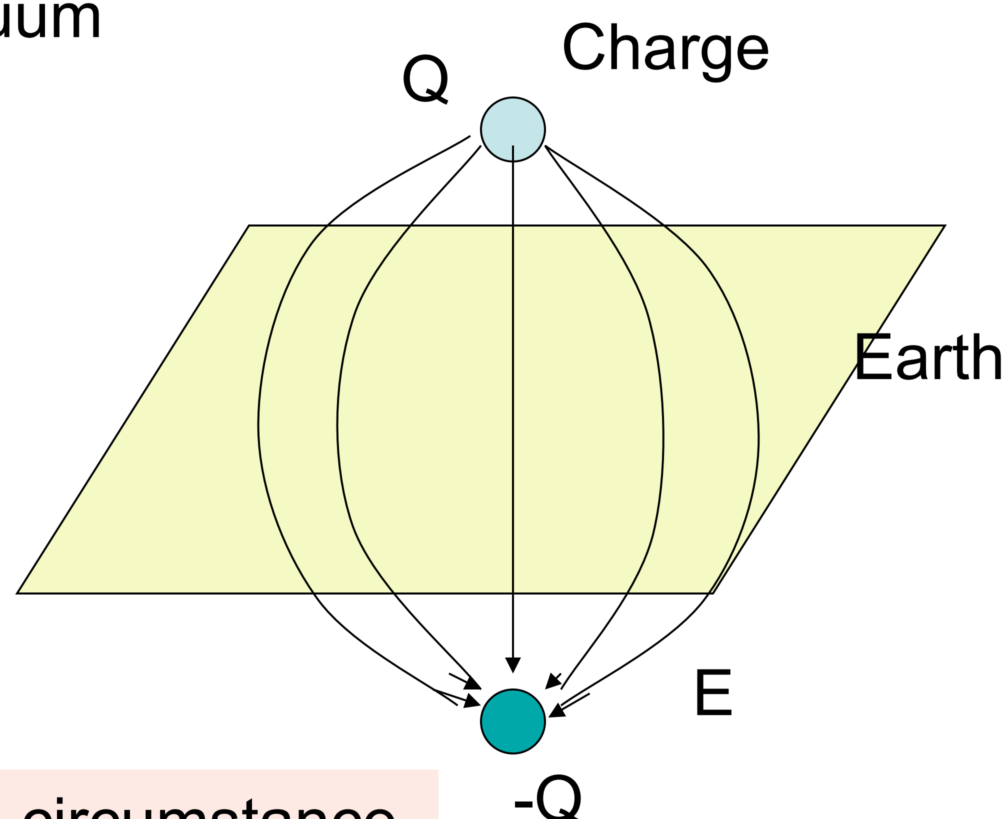


Sato-Toki symmetrization

What is noise??

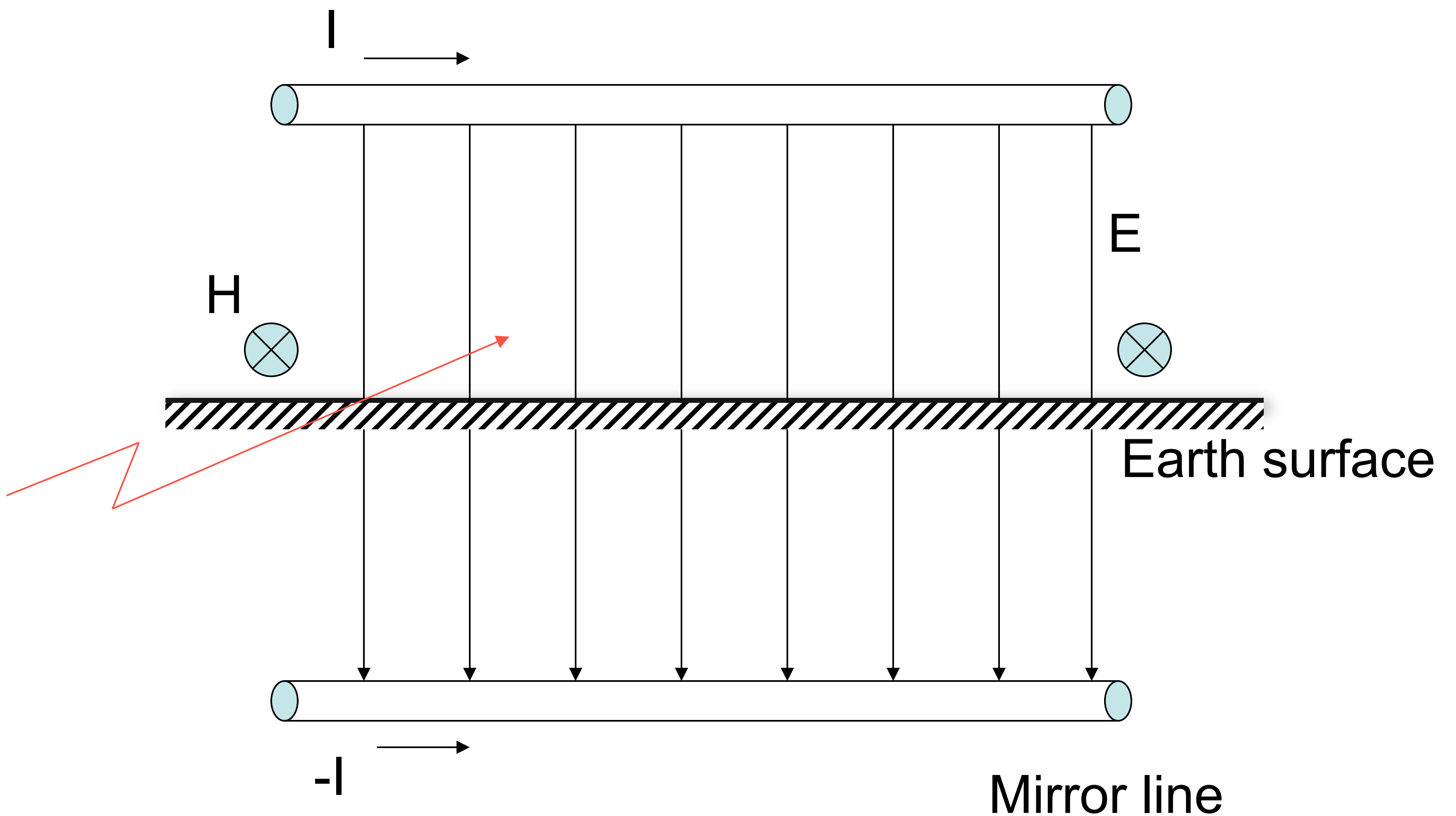
Electric charge (Circumstance)





A charge gets influenced from circumstance.

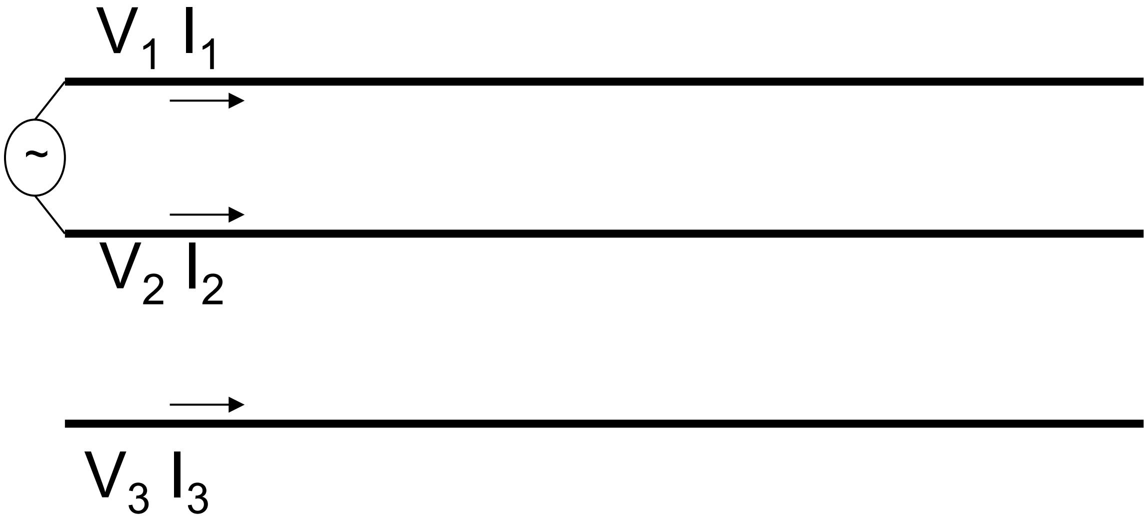
Current in circumstance



Electric and magnetic fields are produced between two lines

3 conductor transmission lines

2 main lines



$I_1+I_2+I_3=0$

Circumstance line
(Noise)

I_1-I_2 = Normal mode （Usually considered）

I_1+I_2= Common mode (Usually neglected)

Usually two modes are coupled.

We should treat these two modes simultaneously.

Multiconductor transmission-line theory

C.R. Paul Book 'Multiconductor transmission-line theory'
(780 pages)

Electric capacity C

$$Q_i(x,t) = \sum_j^N C_{ij} V_j(x,t)$$

Charge Potential

$$\frac{\partial I_i(x,t)}{\partial x} = -\sum_j^N C_{ij} \frac{\partial V_j(x,t)}{\partial t}$$

Coefficient of inductance L

$$\Phi_i(x,t) = \sum_j^N L_{ij} I_j(x,t)$$

Magnetic flux Current

$$\frac{\partial V_i(x,t)}{\partial x} = -\sum_j^N L_{ij} \frac{\partial I_j(x,t)}{\partial t}$$

I,*j*=1…*N* are numbering of transmission lines.

Transmission line equation written in any textbook (N=2)

New transmission-line theory

Toki-Sato theory

1. We reverse one equation

Coefficient of potential *P*

$$P = C^{-1}$$

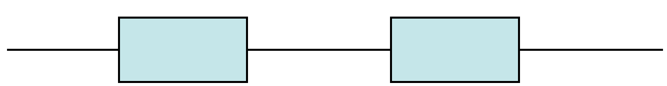
$$\frac{\partial V_i(x,t)}{\partial t} = -\sum_j^N P_{ij} \frac{\partial I_j(x,t)}{\partial x}$$

Normal mode

$$V_n = V_1 - V_2 \qquad I_n = \frac{1}{2}(I_1 - I_2)$$

Common mode

$$V_c = \frac{1}{2}(V_1 + V_2) - V_3 \qquad I_c = I_1 + I_2 = -I_3$$



$$L = L_1 + L_2$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow P = P_1 + P_2$$

Superposition principle

Coupled differential equations

$$\frac{\partial V_n}{\partial t} = -P_n \frac{\partial I_n}{\partial x} - P_{nc} \frac{\partial I_c}{\partial x}$$

$$\frac{\partial V_c}{\partial t} = -P_{cn} \frac{\partial I_n}{\partial x} - P_c \frac{\partial I_c}{\partial x}$$

$$\begin{aligned} P_n &= P_{11} + P_{22} - 2P_{12} \\ P_{nc} &= \frac{1}{2}(P_{11} - P_{22}) - P_{13} + P_{23} \\ P_c &= \frac{1}{4}(P_{11} + P_{22} + 2P_{12} - 4P_{13} - 4P_{23} + 4P_{33}) \end{aligned}$$

$$\begin{aligned} \frac{\partial V_n}{\partial x} &= -L_n \frac{\partial I_n}{\partial t} - L_{nc} \frac{\partial I_c}{\partial t} \\ \frac{\partial V_c}{\partial x} &= -L_{cn} \frac{\partial I_n}{\partial t} - L_c \frac{\partial I_c}{\partial t} \end{aligned}$$

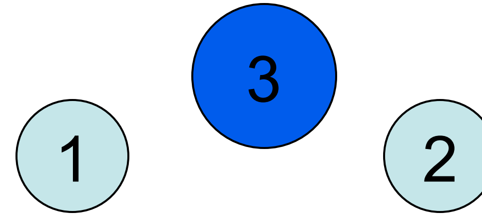
$$\begin{aligned} L_n &= L_{11} + L_{22} - 2L_{12} \\ L_c &= \frac{1}{4}(L_{11} + L_{22} + 2L_{12} - 4L_{13} - 4L_{23} + 4L_{33}) \\ L_{nc} &= \frac{1}{2}(L_{11} - L_{22}) - L_{13} + L_{23} = L_{cn} \end{aligned}$$

$$C = P^{-1}$$

Simple equation

Coefficient of potential（P）made calculation easy.

Symmetric arrangement



$P_{11}=P_{22}$ and $P_{13}=P_{23}$ (same size and same distance)

Decoupling of normal and common modes.

$$\frac{\partial V_n}{\partial t} = -P_n \frac{\partial I_n}{\partial x}$$
$$\frac{\partial V_n}{\partial x} = -L_n \frac{\partial I_n}{\partial t}$$

$$\frac{\partial^2 V_n}{\partial x^2} = \frac{L_n}{P_n}\frac{\partial^2 V_n}{\partial t^2} = L_n C_n \frac{\partial^2 V_n}{\partial t^2}$$

(Textbook equation)

$$\frac{\partial V_c}{\partial t} = -P_c \frac{\partial I_c}{\partial x}$$
$$\frac{\partial V_c}{\partial x} = -L_c \frac{\partial I_c}{\partial t}$$

$$\frac{\partial^2 V_c}{\partial x^2} = \frac{L_c}{P_c}\frac{\partial^2 V_c}{\partial t^2} = L_c C_c \frac{\partial^2 V_c}{\partial t^2}$$

Propagate with light velocity

$$\frac{L}{P} = \varepsilon\mu = \frac{1}{c^2}$$

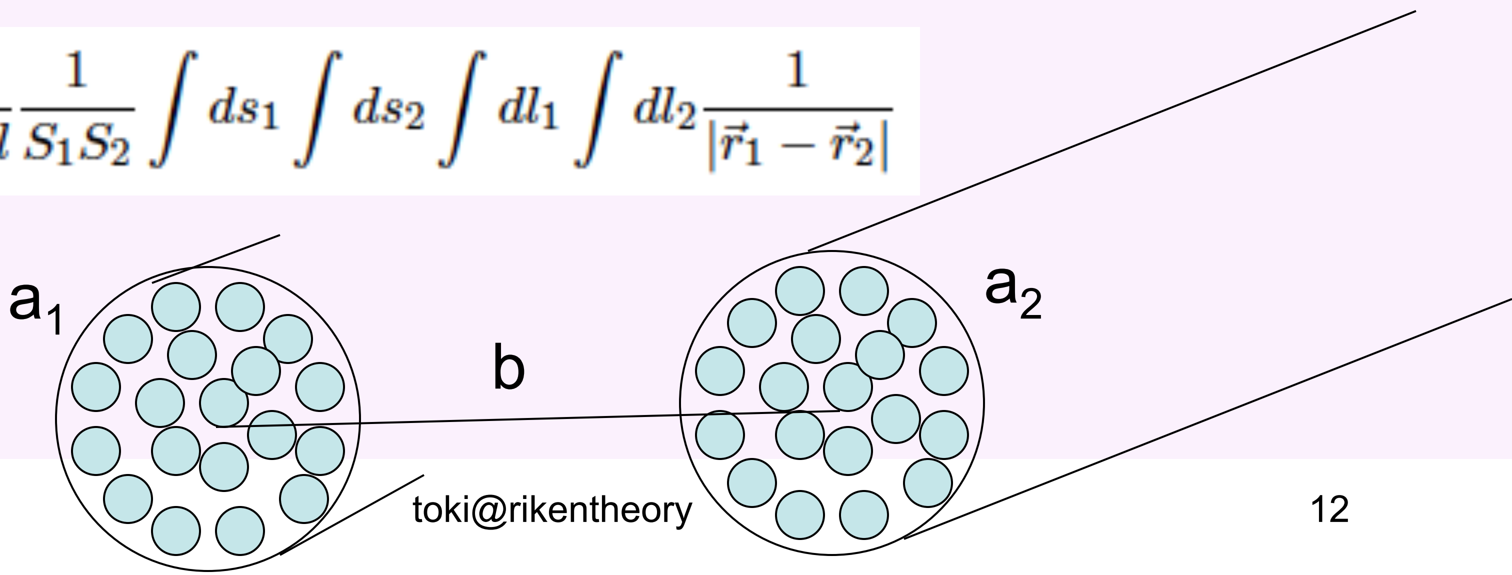
2. Coefficients of potential and inductance

$$V_Q(r) = \frac{1}{4\pi\epsilon}\frac{Q_1 Q_2}{r}$$ Coulomb law → P

$$V_I(r) = \frac{\mu}{4\pi}\frac{I_1 I_2}{r}$$ Ampere law → L

Neumann's formula

$$L_{21} = \frac{\mu}{4\pi l}\frac{1}{S_1 S_2}\int ds_1 \int ds_2 \int dl_1 \int dl_2 \frac{1}{|\vec{r}_1 - \vec{r}_2|}$$



Coefficient of inductance

$$
\begin{aligned}
L_{21} &= \frac{\mu}{2\pi}\frac{1}{S_1 S_2}\int ds_1 \int ds_2 (ln\frac{2l}{|\vec{r}_{1s}-\vec{r}_{2s}|}-1) \\
&= \frac{\mu}{2\pi}\frac{1}{S_1 S_2}\int\int(\ln 2l - \ln|\vec{r}_{1s}-\vec{r}_{2s}| - 1)ds_1 ds_2 \\
&= \frac{\mu}{2\pi}(\ln 2l - 1 - \frac{1}{S_1 S_2}\int\int \ln|\vec{r}_{1s}-\vec{r}_{2s}|ds_1 ds_2)
\end{aligned}
$$

Geometrical mean distance (GMD)

$$\ln\tilde{b} = \frac{1}{S_1 S_2}\int\int \ln|\vec{r}_{1s}-\vec{r}_{2s}|ds_1 ds_2$$

$$L_{21} = \frac{\mu}{2\pi}(\ln 2l - 1 - \ln\tilde{b}) = \frac{\mu}{2\pi}(\ln\frac{2l}{\tilde{b}} - 1)$$

Coefficient of potential

Charge distribution is same as the current distribution

$$\frac{\partial I(x,t)}{\partial x} = -\frac{\partial Q(x,t)}{\partial t}$$

Neumann's formula

$$P_{21} = \frac{1}{4\pi\epsilon l}\frac{1}{S_1 S_2}\int ds_1 \int ds_2 \int dl_1 \int dl_2 \frac{1}{|\vec{r}_1 - \vec{r}_2|}$$

$$P_{21} = \frac{1}{2\pi\epsilon}(\ln\frac{2l}{\tilde{b}} - 1)$$

P and L have the same geometrical expression

$$\frac{P_{ij}}{L_{ij}} = \frac{1}{\varepsilon\mu} = c^2$$

Characteristic impedance

$$P_{ij}L_{ij} = Z_{ij}^2$$

Three Conductor Transmission Line Theory and Origin of Electromagnetic Radiation and Noise

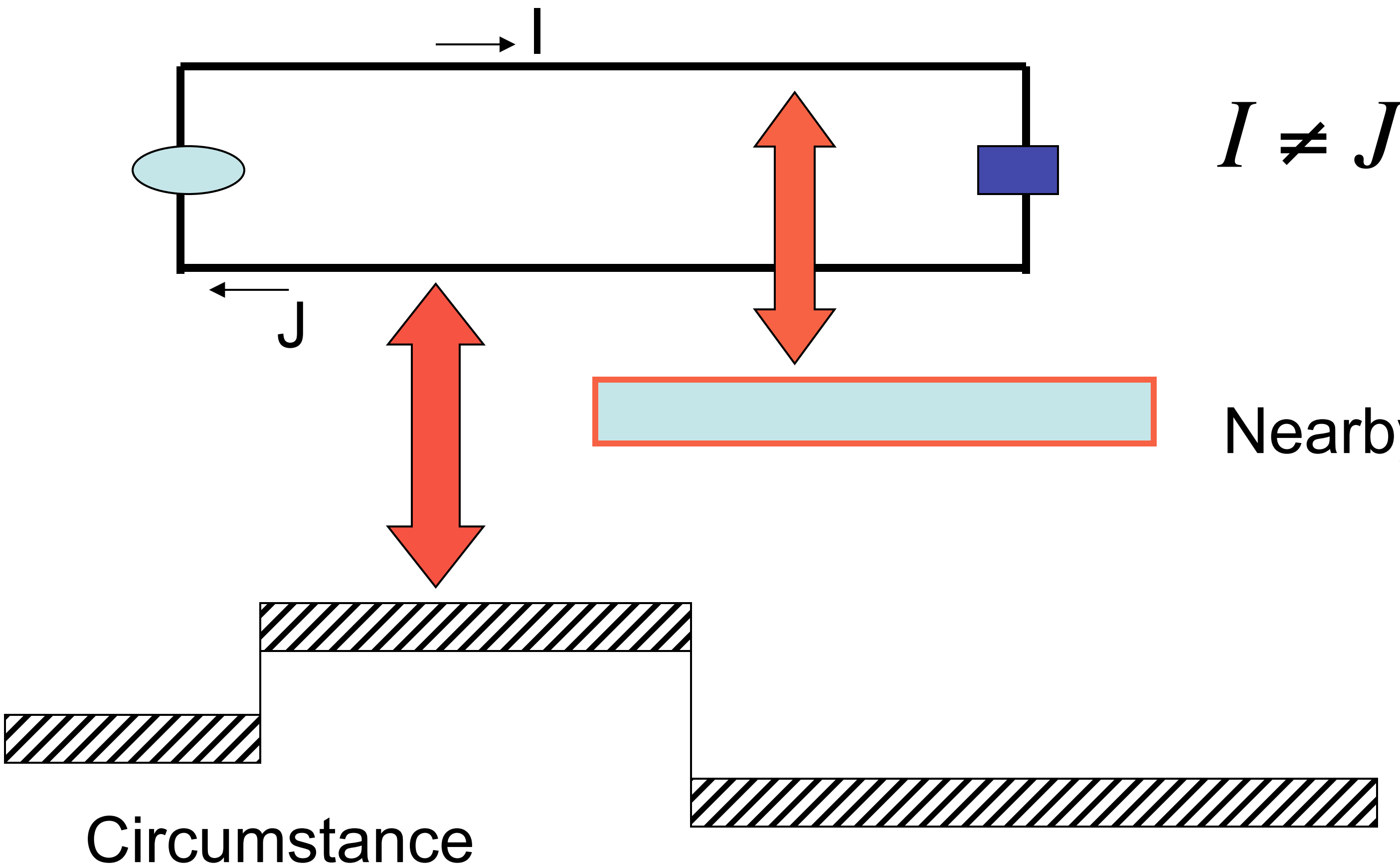
Hiroshi Toki[a)]* and Kenji Sato[a,b)]†

JPSJ 78 (2009)094201
Journal of physical Society of Japan

- Normal and common modes usually couple.
- Common mode is the source of noise.
- To decouple the two modes, two lines should have the same size and same distance from circumstance.
- However, it is hard usually to satisfy this condition.

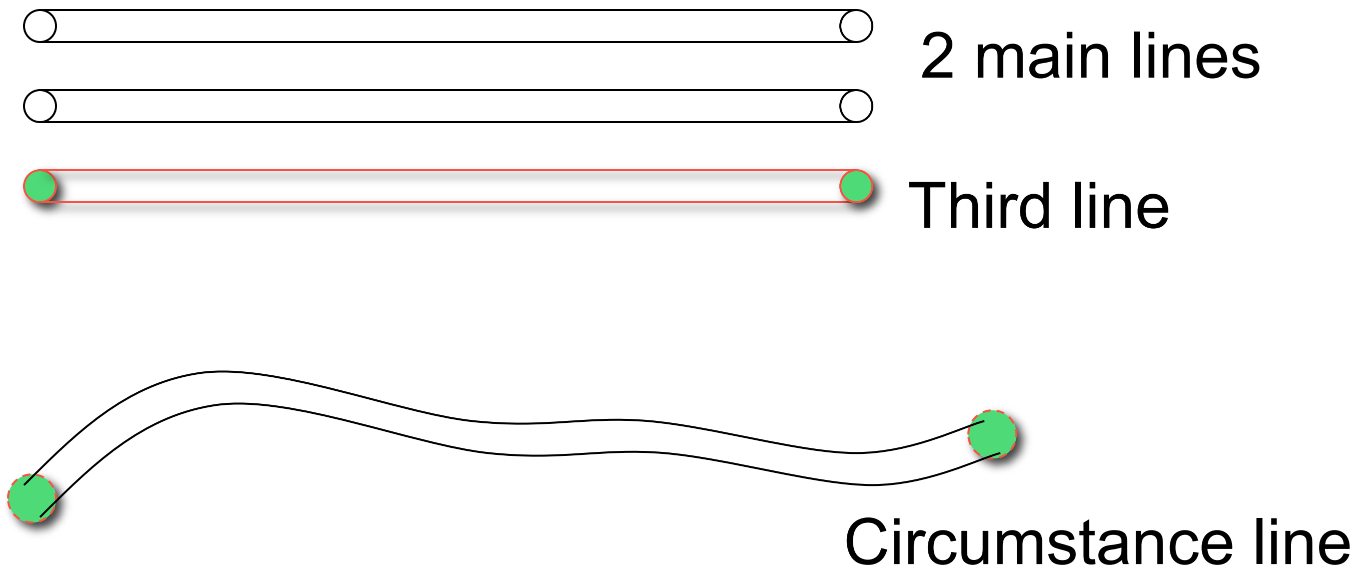
Source of noise

Common mode is a problem.



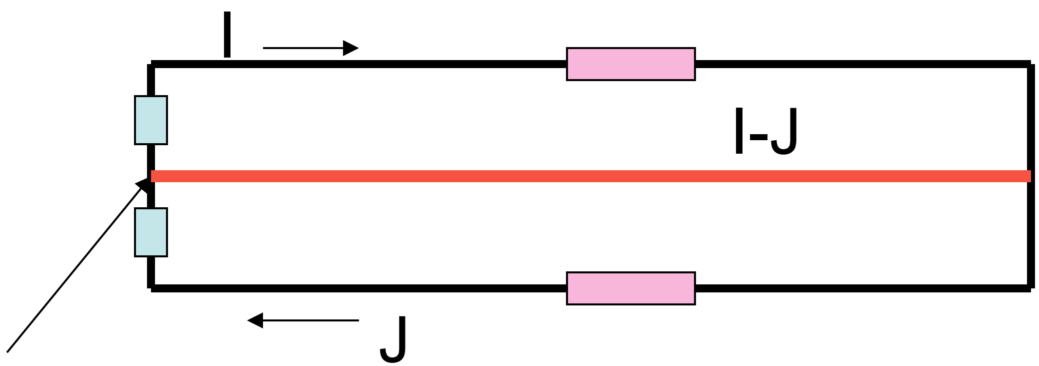
How to decouple from the circumstance?

Introduce one more line and make the influence from the circumstance minimized.



We can verify then that a symmetrized three line system makes the normal mode decouples from the circumstance.

Confine EM fields in a circuit



Middle line

Connect here

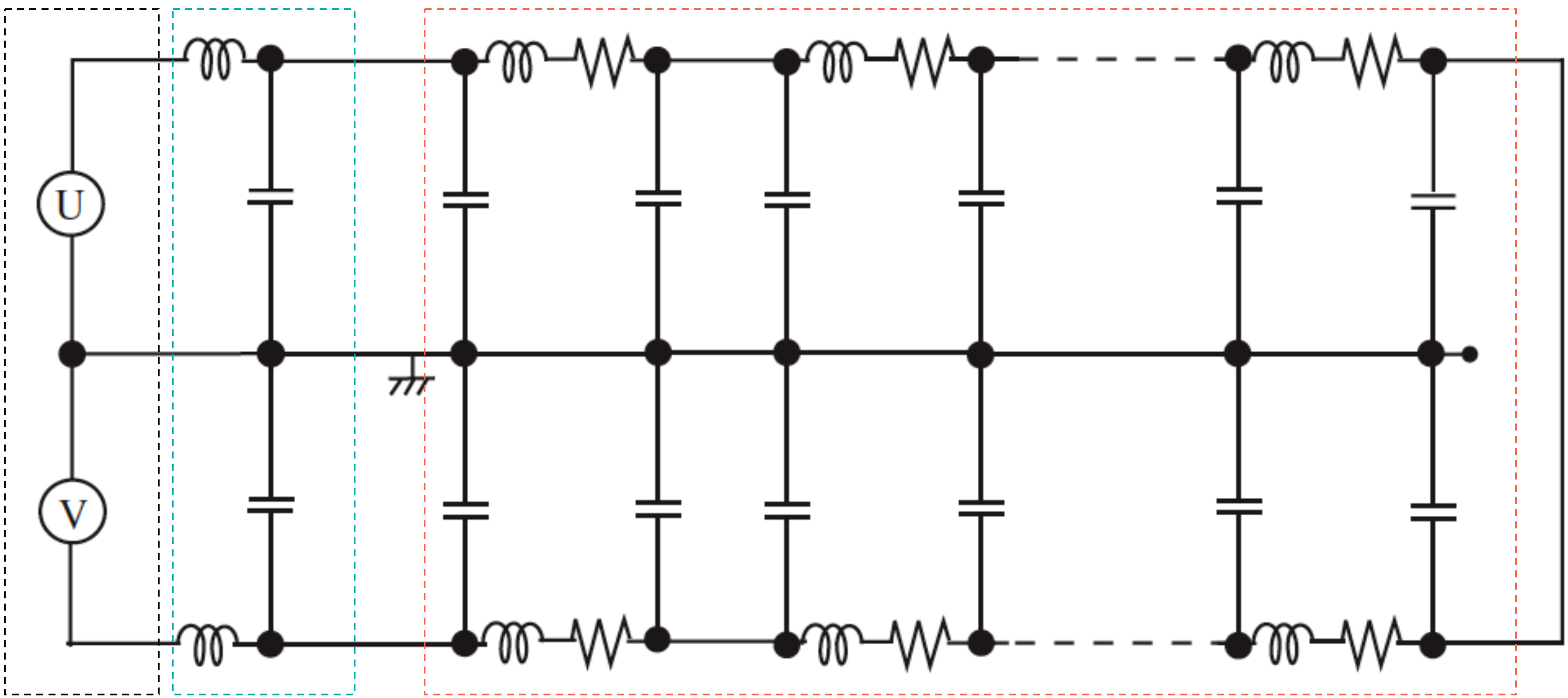
The middle line is a part of circuit.

Arrange electric loads symmetrically around the middle line.
（Decouple normal and common modes.）

Sato-Toki; NIM(2006)

HIMAC method（Sato-Toki circuit）

Symmetrization



Power

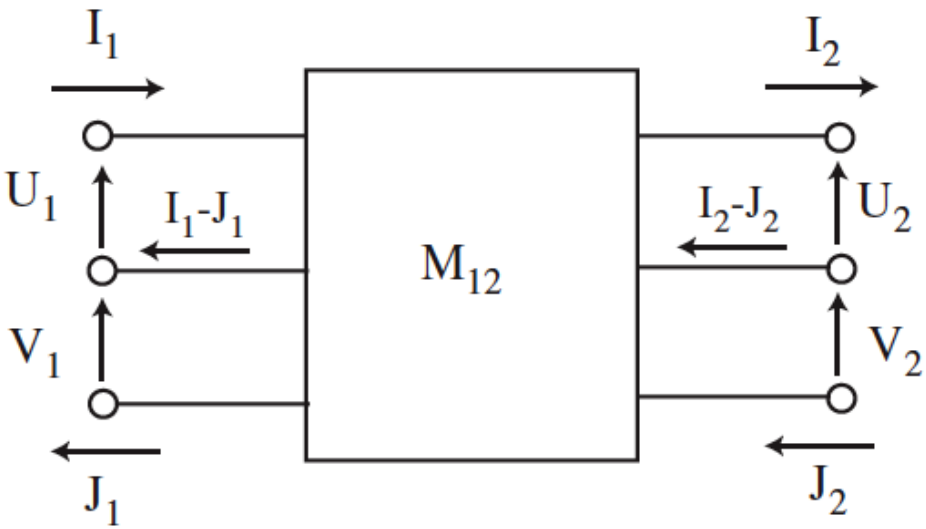
Noise filter

Magnet (Load)

ppm noise level

Synchrotron magnet power supply network with normal and common modes including noise filtering

K. Sato, H. Toki* NIM A565 (2006) 351



3 line lumped circuit

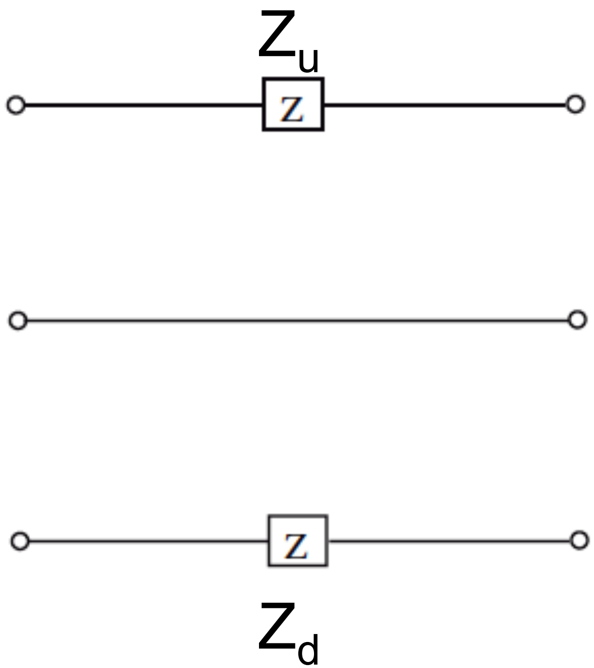
$$\begin{pmatrix} U_1 \\ V_1 \\ I_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} U_2 \\ V_2 \\ I_2 \\ J_2 \end{pmatrix}.$$

Normal mode （We calculate this part）

Common mode（We do not handle this part）

$$\begin{pmatrix} U_1+V_1 \\ I_1+J_1 \\ U_1-V_1 \\ I_1-J_1 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \begin{pmatrix} U_2+V_2 \\ I_2+J_2 \\ U_2-V_2 \\ I_2-J_2 \end{pmatrix}.$$

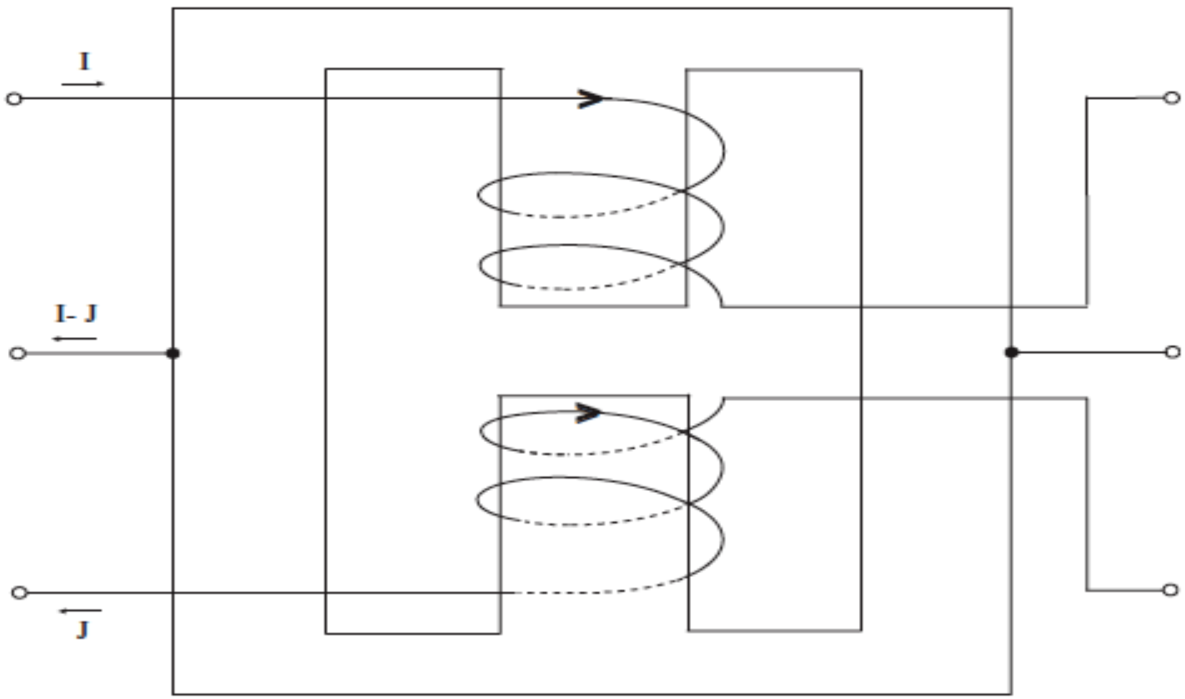
Normal and common modes decouple with symmetric arrangement.



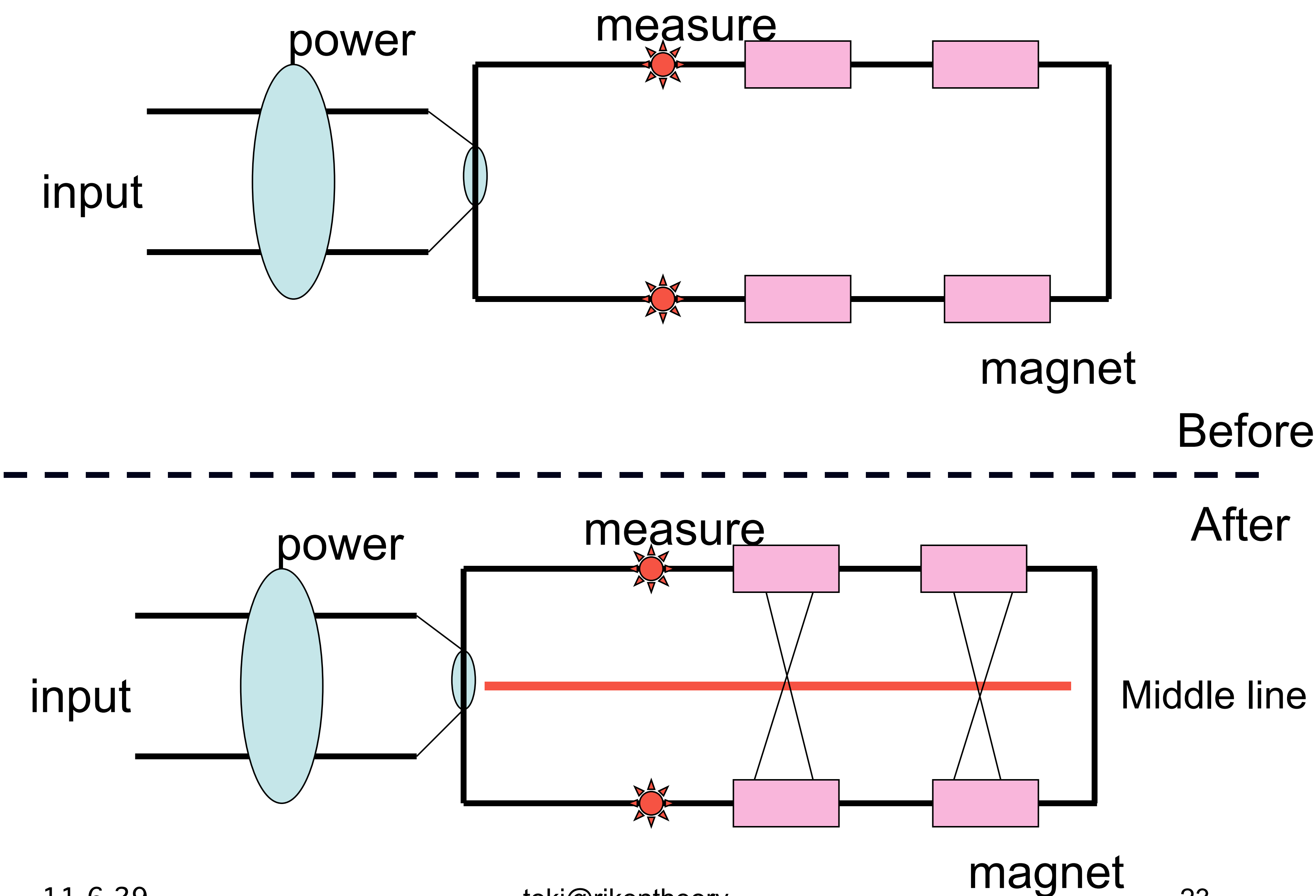
$$\begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} = \begin{pmatrix} 1 & (Z_u + Z_d)/2 & 0 & (Z_u - Z_d)/2 \\ 0 & 1 & 0 & 0 \\ 0 & (Z_u - Z_d)/2 & 1 & (Z_u + Z_d)/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

When $Z_u = Z_d$, the non-diagonal term becomes zero.

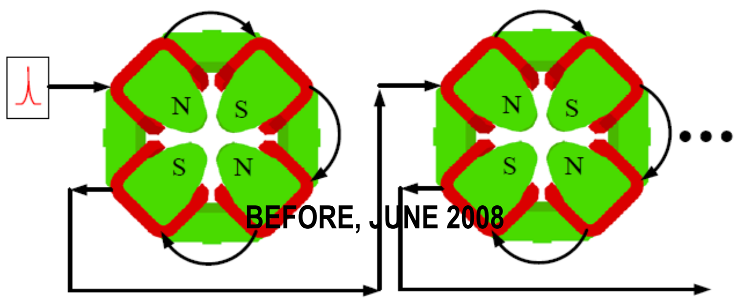
Normal mode (I+J) magnet



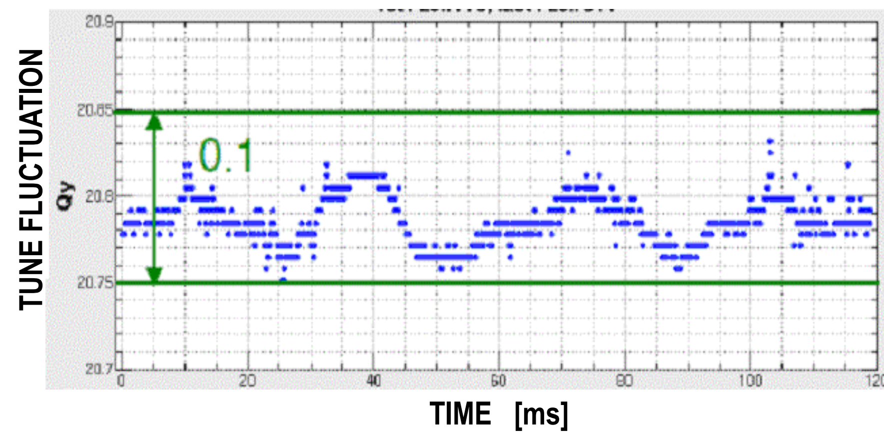
Common mode (I-J) is not used.

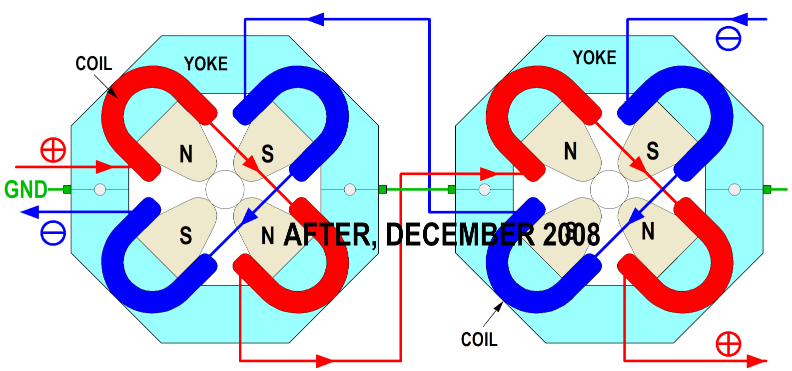

measure
power
input
magnet
Before
After
measure
power
input
Middle line
magnet

配線の対称化と同極接続:結果

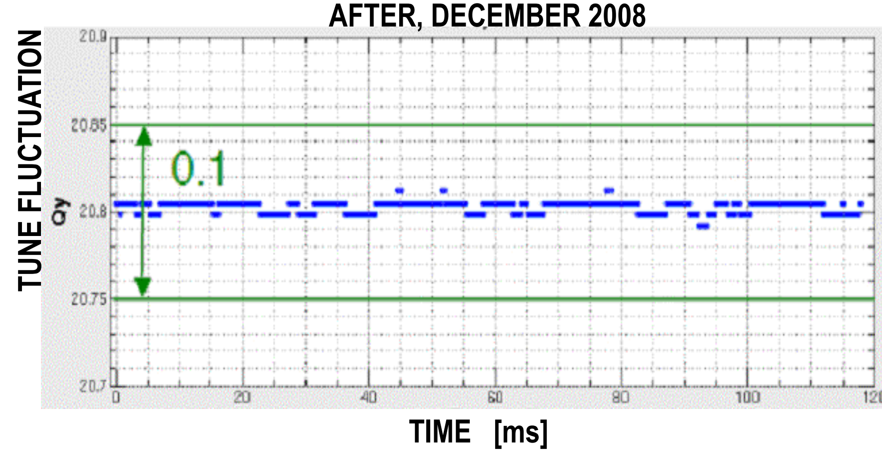


BEFORE, JUNE 2008





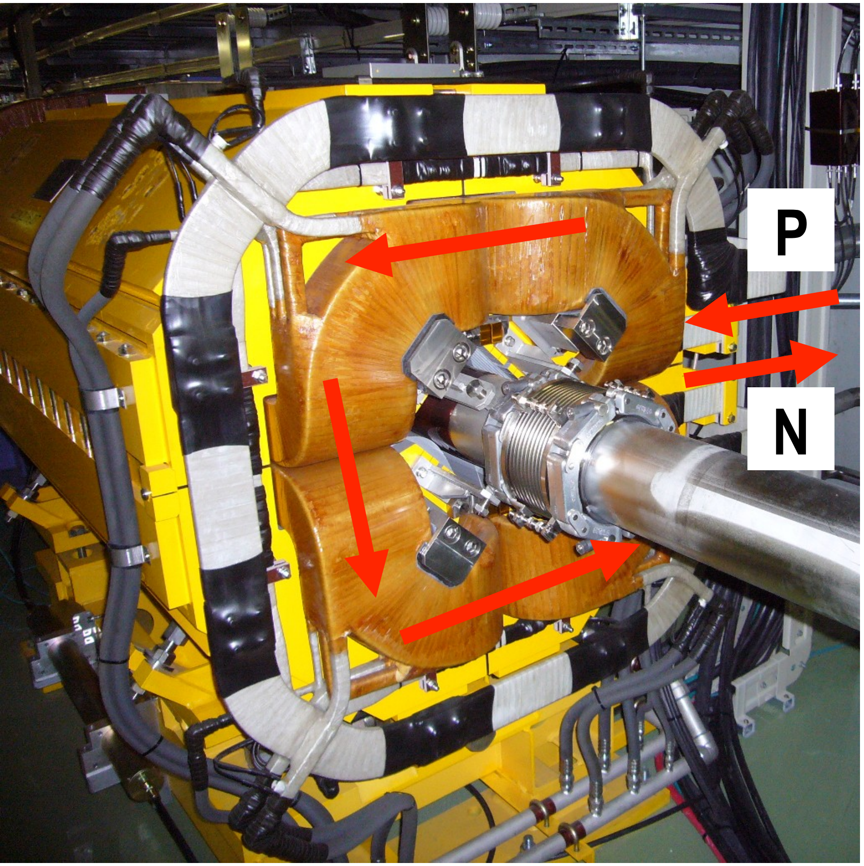
AFTER, DECEMBER 2008

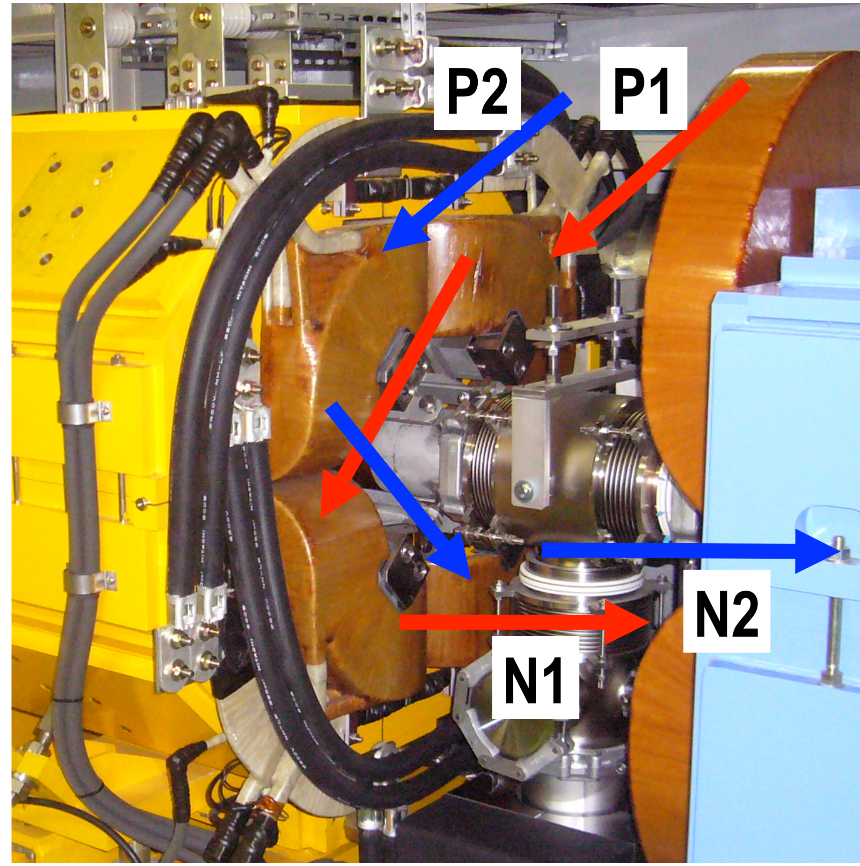


負荷配線の対称化

QMの場合

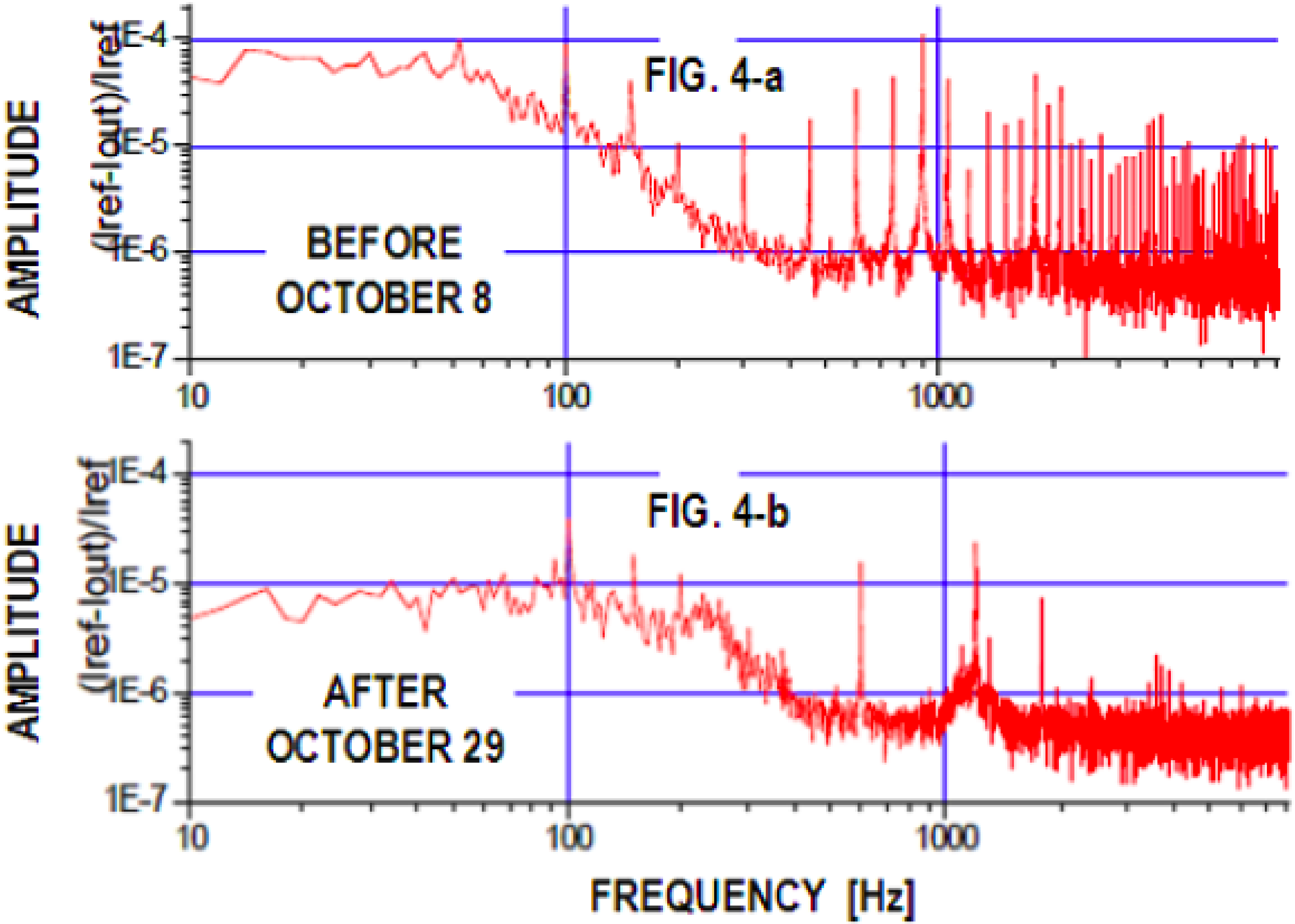
磁極間がブスで接続されていたため、ブスを取り外し、ケーブルで接続



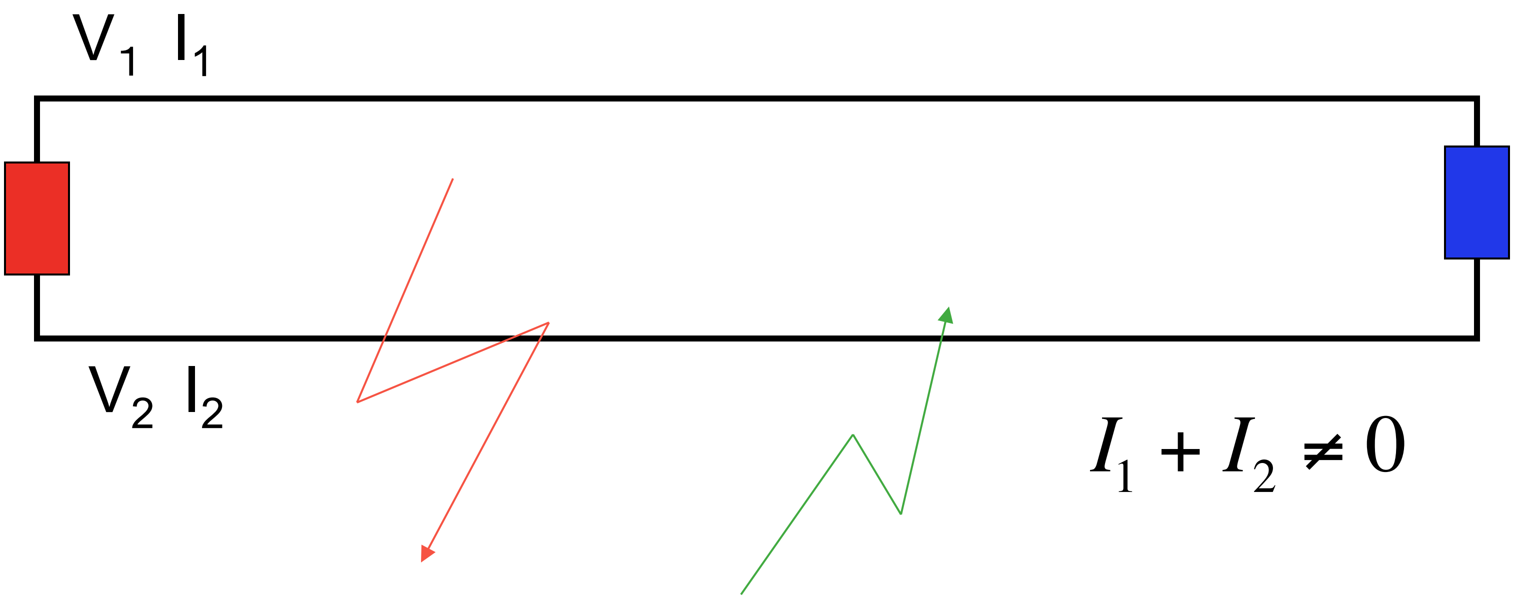


（1ヶ月で実施）

Physics of symmetrization of J-PARC (MR)



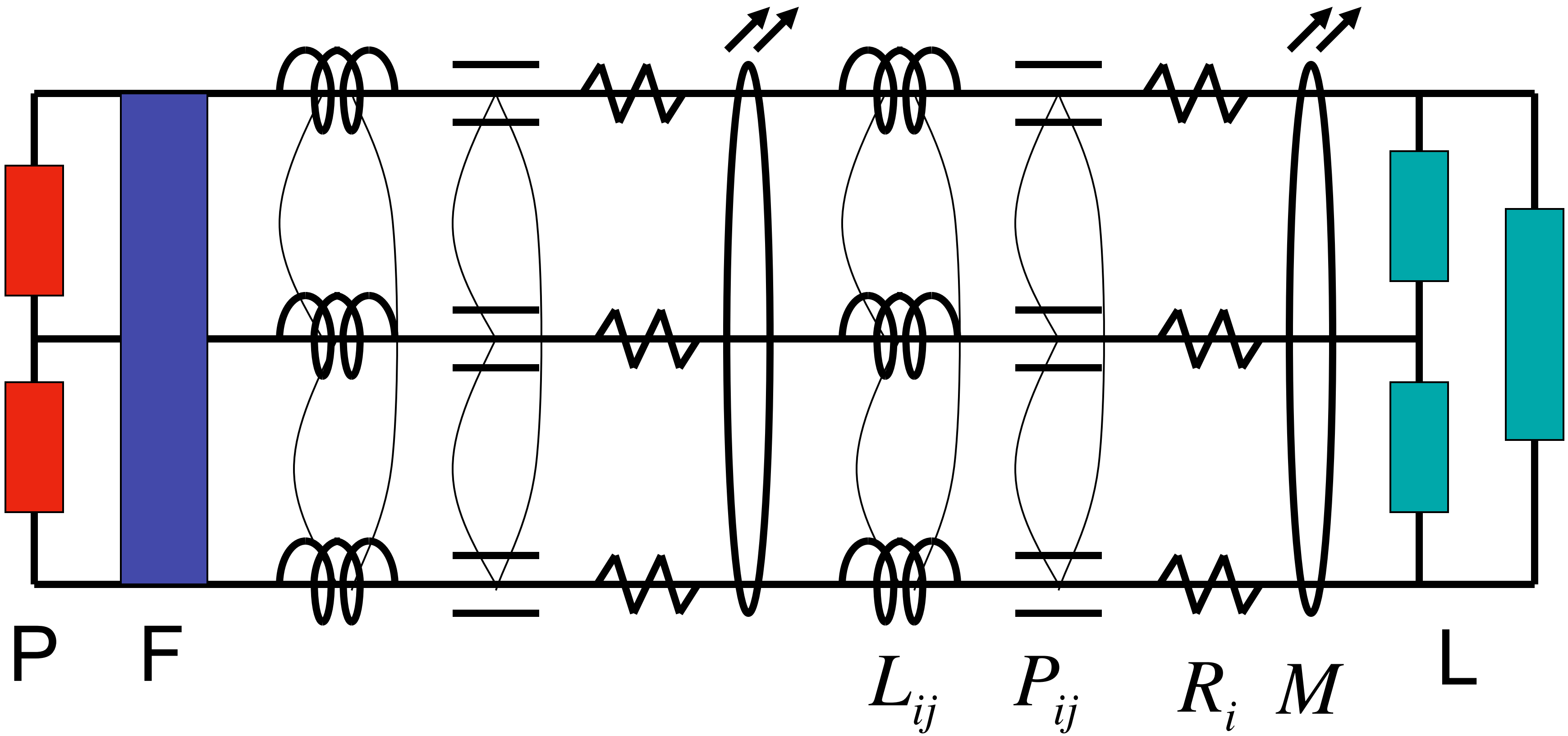
Include EM radiation



$$I_1 + I_2 \neq 0$$

- We can work out new multiconductor transmission-line equation with radiation starting from Maxwell equation.
- Again the Sato-Toki symmetrization decouples the normal mode from the common and antenna modes.

Noiseless electric circuit（Noise is EM wave）



Transmission-lines have P, L, R, M effects

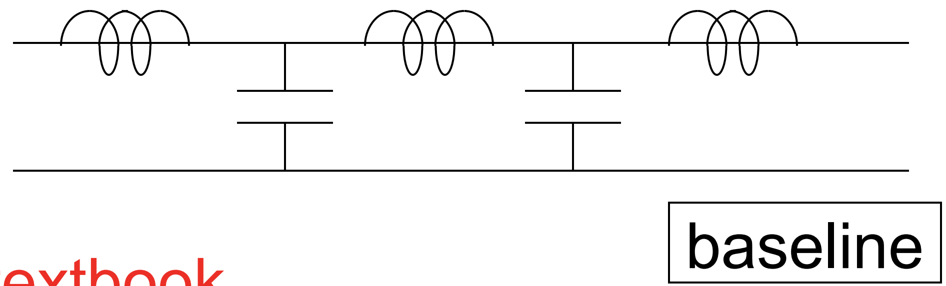
History of Transmission-line theory

- Kirchhoff (1857) : Electric transport equation (1 line)

$$I = -4\gamma \frac{l}{r}\left(\frac{\partial Q}{\partial x} + \frac{2}{c_W^2}\frac{\partial I}{\partial t}\right) \qquad \frac{\partial I}{\partial x} = -\frac{\partial Q}{\partial t} \qquad \gamma = \ln\frac{l}{\alpha}$$

- Heviside(1876:1886) : 2 lines and use C

$$\gamma = \ln\frac{d}{\alpha}$$

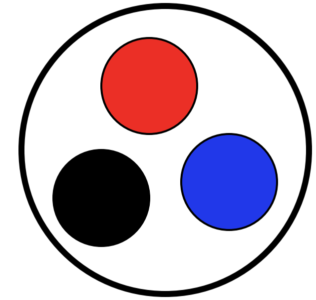


- Maxwell (1873) wrote a textbook

  Discuss both P and C
  V_1+V_2 is meaningless and take C

$$V_i = \sum_j P_{ij} Q_j \qquad Q_i = \sum_j C_{ij} V_j$$

Conclusion



- 3-conductor transmission-line theory
- We calculate coefficients of coupled differential equations using P and L.
- Usual electric circuit couples always with circumstance.
- 3 line electric circuit with symmetric arrangement makes noise free circuit.

(Sato-Toki symmetrization)