Spontaneous breaking of supersymmetry

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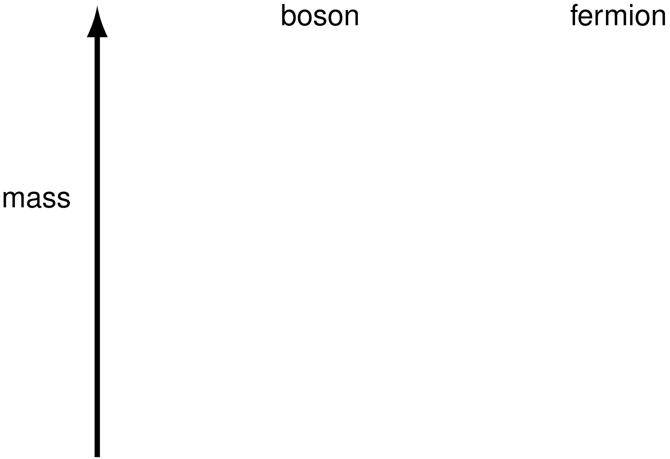
SUperSYmmetry (SUSY) postulates

invariance under the “exchange” of these bosons and fermions!

But our real world is not supersymmetric. . .

- Mass spectrum of elementary particles



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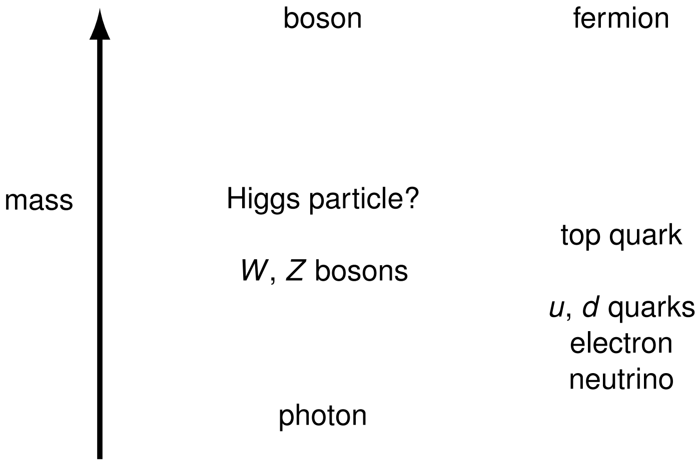

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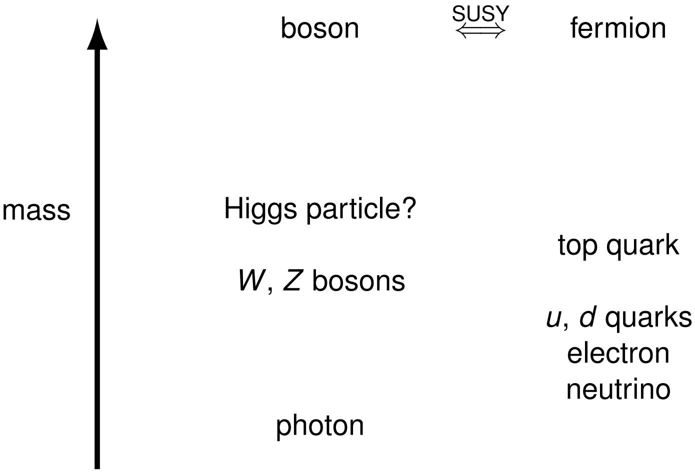

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Higgs particle?	$\Longleftrightarrow$	
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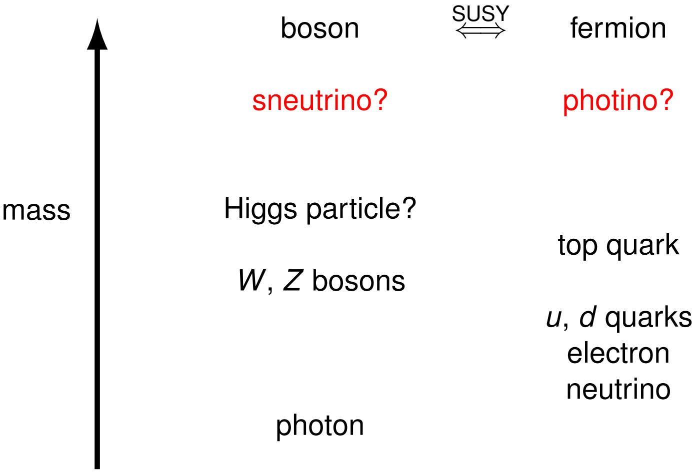

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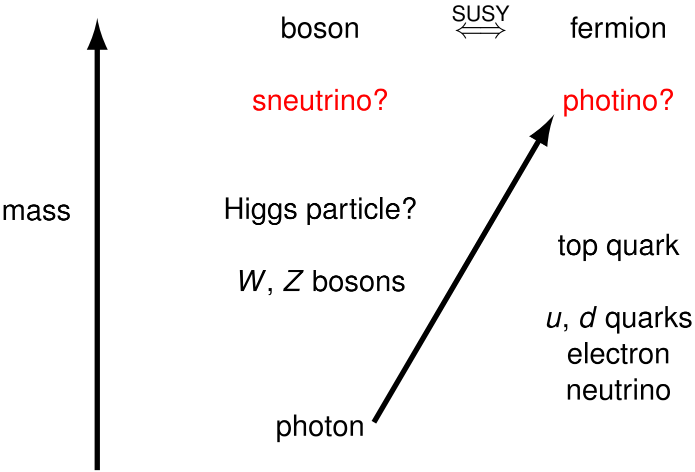

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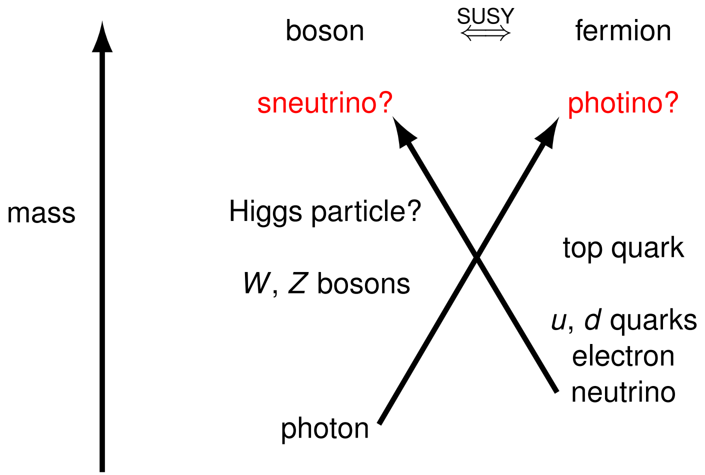

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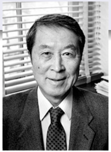
Nambu-Goldstone theorem

When ordinary (continuous) symmetry is spontaneously broken, there emerge massless boson(s)

- Chiral symmetry is spontaneously broken! $\Rightarrow$ pions



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SUSY and Quantum Field Theory (QFT)

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$$\phi(\boldsymbol{x}, t), \qquad \boldsymbol{x} \in \mathbb{R}^3, \quad t\text{: time}$$

- This is technically complicated, so let us consider QFT in an artificial space, consisting of a single point

$$\phi(*, t) \equiv q(t), \qquad t\text{: time}$$

This is quantum mechanics of a single degree of freedom...

SUSY Quantum Mechanics (QM)

- Hamiltonian of SUSY QM (obtained by $p \to -i\hbar\frac{d}{dq}$)

$$H = -\frac{\hbar^2}{2}\frac{d^2}{dq^2} + \frac{1}{2}W'(q)^2 + \hbar W''(q)\left(b^\dagger b - \frac{1}{2}\right),$$

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- Among E, the smallest E ($\equiv E_0$), the ground state energy will be very important in what follows

Creation and annihilation operators of a fermion

- $b^\dagger$ and b obey the relations

$$bb^\dagger + b^\dagger b = 1, \qquad (b^\dagger)^2 = b^2 = 0$$

and, from these,

$$|0\rangle \xrightarrow{b^\dagger} |1\rangle \xrightarrow{b^\dagger} 0$$
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- These can be represented in terms of a “two-story” notation

$$|0\rangle \Leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{bosonic}, \qquad |1\rangle \Leftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{fermionic}$$

and 2×2 matrices

$$b \Leftrightarrow \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \qquad b^\dagger \Leftrightarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

SUSY QM

- In this two-story notation,

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- Now, the fundamental relation in SUSY QM is

$$H = Q^2$$

where Q is an (hermitian) operator called the super-charge

$$Q = \frac{1}{\sqrt{2}}\left[-i\hbar\frac{d}{dq}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} + W'(q)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\right]$$

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- The supercharge maps the boson into the fermion and vice versa

$$Q \begin{pmatrix} 0 \\ * \end{pmatrix} = \begin{pmatrix} * \\ 0 \end{pmatrix}, \qquad Q \begin{pmatrix} * \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ * \end{pmatrix}$$

and generates SUSY transformation

Basic property of SUSY QM (I)

- Since

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the energy eigenvalue E

$$H\Psi(q) = Q^2\Psi(q) = E\Psi(q)$$

is positive or zero

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$$E = 0 \rightarrow Q^2\Psi(q) = 0 \rightarrow Q\Psi(q) = 0$$

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Basic property of SUSY QM (II)

- For any positive energy eigenvalue $E > 0$

$$H\Psi(q) = E\Psi(q), \qquad E > 0,$$

one may always apply Q to produce another state $\Phi(q)$

$$\Phi(q) = Q\Psi(q)$$

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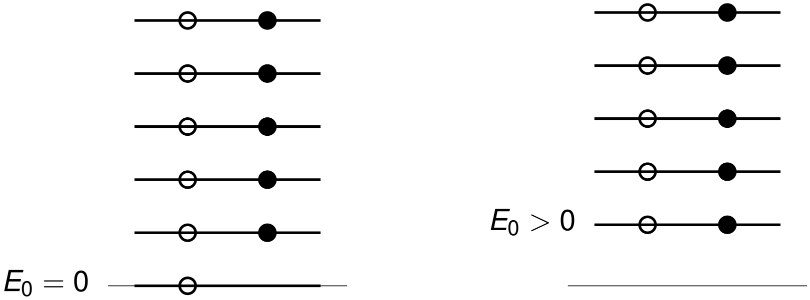
- N.B. The pairing is impossible for $E = 0$ because $Q\Psi(q) = 0$

Possible patterns of the energy spectrum



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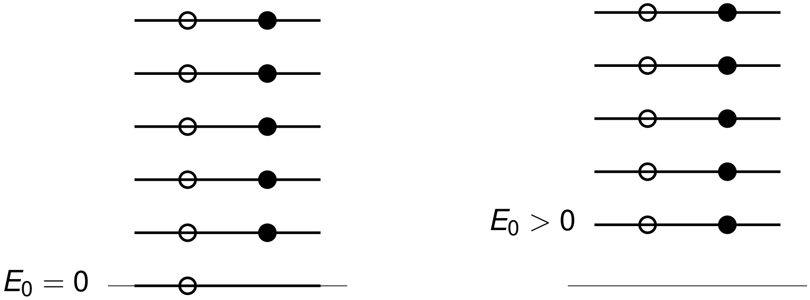

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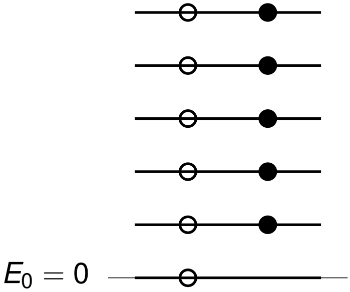

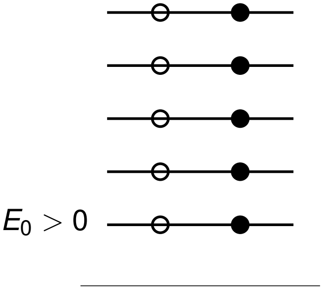

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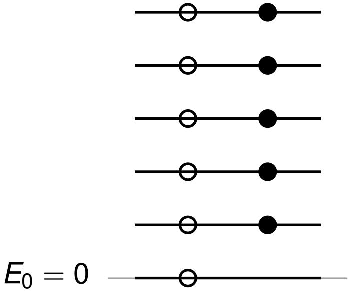
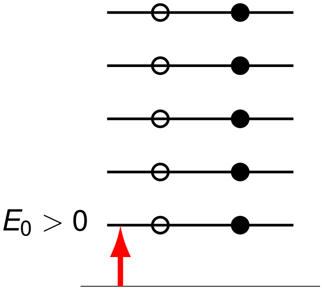
SUSY is spontaneously broken!





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Zero-energy $E = 0$ state in SUSY QM

- Zero-energy $E = 0$ state satisfies the 1st order differential eq.

$$Q\Psi(q) = -\frac{i\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\left[\frac{d}{dq} + \frac{1}{\hbar}W'(q)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right]\Psi(q) = 0$$

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- The solution, however, must be normalizable

$$\int_{-\infty}^{\infty} dq \, \Psi(q)^\dagger \Psi(q) < \infty$$

This requires

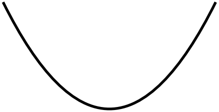
$$W(q) \to +\infty \quad \text{for} \quad q \to \pm\infty$$

or

$$W(q) \to -\infty \quad \text{for} \quad q \to \pm\infty$$

Asymptotic behavior of $W(q)$ determines which. . .

- If $W(q)$ behaves as



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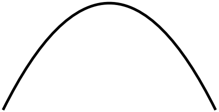
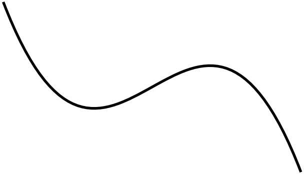


There exists $E = 0$ state and SUSY is not broken

- Otherwise, for example,



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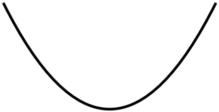
Let us consider two examples

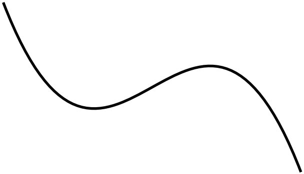
- Model I

$$W(q) = \frac{1}{2}\omega q^2$$



- Model II

$$W(q) = \frac{1}{2}\omega q^2 - \frac{1}{3}\lambda q^3$$



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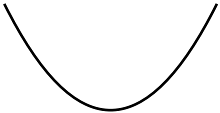

SUSY will not be spontaneously broken

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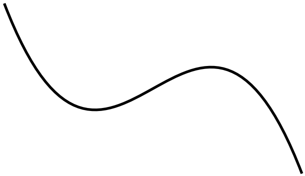

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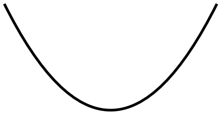

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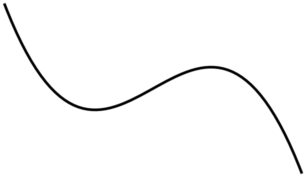


SUSY will be spontaneously broken

The ground state in model I

- The Hamiltonian

$$H = \left(-\frac{\hbar^2}{2}\frac{d^2}{dq^2} + \frac{1}{2}\omega^2 q^2\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{1}{2}\hbar\omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

  represents two independent harmonic oscillators with shifted zero-point energies $\pm(1/2)\hbar\omega$: Exactly solvable
- The unique ground state

$$\Psi_0(q) = \begin{pmatrix} 0 \\ \left(\frac{\omega}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{\omega}{2\hbar}q^2\right)\end{pmatrix}$$

  has the energy

$$E_0 = \frac{1}{2}\hbar\omega - \frac{1}{2}\hbar\omega = 0$$

  and is annihilated by the supercharge

$$Q\Psi_0(q) = 0$$

1st excited states in model I

- The first excited states

$$\Psi_1(q) = \begin{pmatrix} 0 \\ \left(\frac{4\omega^3}{\pi\hbar^3}\right)^{1/4} q \exp\left(-\frac{\omega}{2\hbar}q^2\right) \end{pmatrix} \propto Q\Phi_1(q) \qquad \text{bosonic}$$

and

$$\Phi_1(q) = \begin{pmatrix} \left(\frac{\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{\omega}{2\hbar}q^2\right) \\ 0 \end{pmatrix} \propto Q\Psi_1(q) \qquad \text{fermionic}$$

have the degenerate energies

$$E_1 = \frac{3}{2}\hbar\omega - \frac{1}{2}\hbar\omega = \hbar\omega$$

$$E_1 = \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \hbar\omega$$

Spectrum in model I

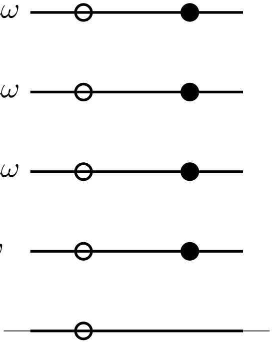
$E_4 = 4\hbar\omega$

$E_3 = 3\hbar\omega$

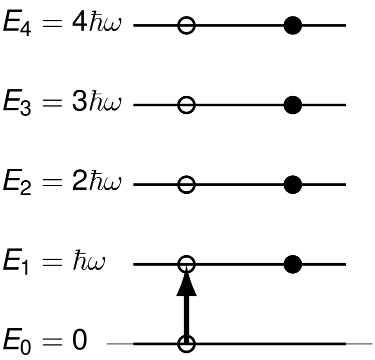
$E_2 = 2\hbar\omega$

$E_1 = \hbar\omega$

$E_0 = 0$



Spectrum in model I



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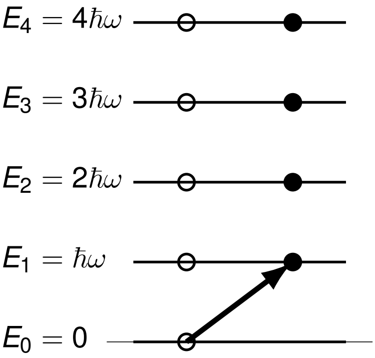

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Model II in the perturbation theory

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$$H = \left[-\frac{\hbar^2}{2} \frac{d^2}{dq^2} + \frac{1}{2} \omega^2 q^2 \left(1 - \frac{\lambda}{\omega} q \right)^2 \right] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \left(\frac{1}{2} \hbar\omega - \hbar\lambda q \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is not exactly solvable for $\lambda \neq 0$

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- Perturbation theory

$$H = H_0 + H'$$

  where

$$H_0 = \left(-\frac{\hbar^2}{2}\frac{d^2}{dq^2} + \frac{1}{2}\omega^2 q^2 \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\hbar\omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \Leftarrow \text{Model I}$$

$$H' = \left(-\lambda\omega q^3 + \frac{1}{2}\lambda^2 q^4 \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \hbar\lambda q \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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$$E_0^{(0)} = \frac{1}{2}\hbar\omega - \frac{1}{2}\hbar\omega = 0$$

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$$E_0^{(2)} = \frac{3\hbar^2}{8\omega^2}\lambda^2 - \frac{3\hbar^2}{8\omega^2}\lambda^2 = 0$$

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- Is this just a coincidence?

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- Proof: The would-be $E = 0$ wave function

$$\exp\left(-\frac{1}{\hbar}W(q)\right) = \exp\left[-\frac{1}{\hbar}\left(\frac{1}{2}\omega q^2 - \frac{1}{3}\lambda q^3\right)\right]$$

is normalizable to all orders of the power-series expansion to $O(\lambda^N)$

$$\exp\left(-\frac{1}{\hbar}W(q)\right) \simeq \exp\left(-\frac{\omega}{2\hbar}q^2\right)\sum_{n=0}^{N}\frac{1}{n!}\left(\frac{\lambda}{3\hbar}q^3\right)^n .$$

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- Spontaneous SUSY breaking in this system cannot be seen by perturbation theory!

Non-renormalization theorem

- More generally, for any superpotential $W(q)$,

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If $E_0 = 0$ in the classical theory (i.e., when $\hbar = 0$), then $E_0 = 0$ remains in all orders of perturbation theory

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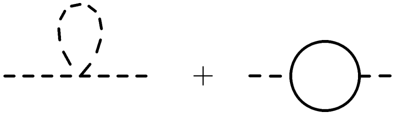
- SUSY theories typically exhibit this sort of remarkable stability under perturbative (or radiative) corrections
- Such stability results from the cancellation between the contribution from bosons and fermions



Semi-classical approximation

- Semi-classical (or WKB, or instanton) approximation yields

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- Spontaneous SUSY breaking in this system is a purely non-perturbative phenomenon!

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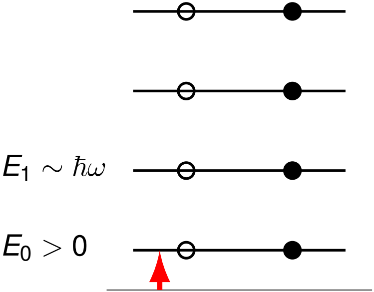

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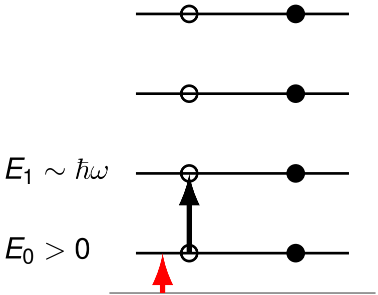

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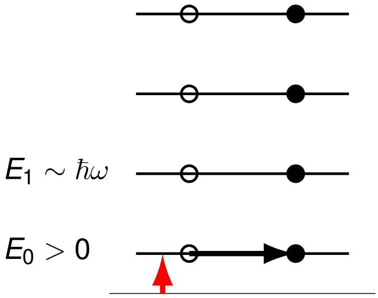

- The excitation energy from the ground state to the first excited bosonic state is $\sim \hbar\omega$
- There is no excitation energy from the ground state to another fermionic ground state! $=$ Nambu-Goldstone theorem

Dictionary from SUSY QM to SUSY QFT

- Dictionary

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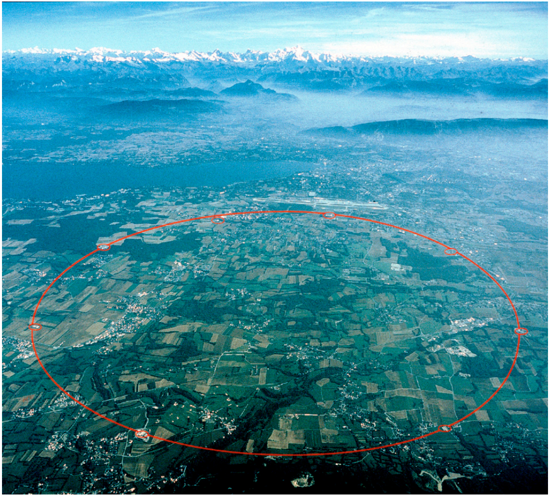
- The cancellation between bosons and fermions makes the divergence of the mass of the Higgs particle quite moderate
- Consistency of string theory requires SUSY

Large Hadron Collider (LHC)

- To find evidence of SUSY is one of the main objectives...



Non-perturbative study of SUSY QFT

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  We do not know the equation, beyond the perturbation theory!

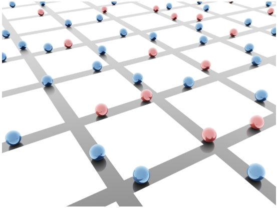
- We only have

  perturbation theory + consistency arguments

Non-perturbative definition by the lattice

- QFT is quantum mechanics of infinitely many variables (field)

$$\phi(\boldsymbol{x}, t), \qquad \boldsymbol{x} \in \mathbb{R}^3, \quad t\text{: time}$$
$$\Downarrow$$
$$\phi(\boldsymbol{x}, t), \qquad \boldsymbol{x} \in \mathbb{Z}^3, \quad t \in \mathbb{Z}$$

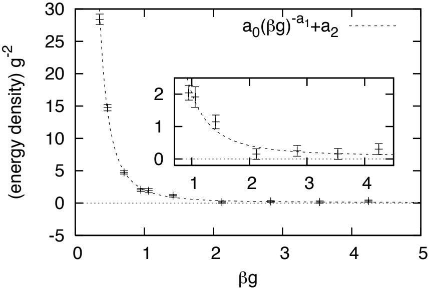


- This discretization however breaks SUSY…

Only quite recently… (I. Kanamori, PRD 79 (2009))

- Vacuum energy density $\mathcal{E}_0$ of a certain SUSY QFT (SUSY Yang-Mills theory) defined in one-dimensional space

$$\mathcal{E}_0/g^2 = 0.09 \pm 0.09(\text{sys})\,{}^{+0.10}_{-0.08}(\text{stat})$$



- it appears that the spontaneous SUSY breaking in this system is unlikely…

Summary

- SUSY is a very interesting possibility, but it must be spontaneously broken to be true in the real world
- To study nonperturbative spontaneous breaking of SUSY from first principles, we need a non-perturbative formulation of SUSY QFT
- This is not yet available, although we had recently a promising success at least partially