Properties of nuclei around proton drip line and many-body effects on mirror asymmetry

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We discuss the mirror asymmetry in light sd-shell nuclei by using recent data on nuclei near the proton drip line. It is clarified that the many-body effects are as important as the single-particle effects. The reduction in the amount of residual interaction matrix elements concerning the $(1s_{1/2})_p$ orbit plays a significant role in the Thomas-Ehrman shifts. By taking this effect into account, the energy levels of the $18 \leq A \lesssim 20$ nuclei are reproduced to a high accuracy by a shell model calculation. Based on this shell model calculation, several key states to the rp-process are pointed out. Analyzing the asymmetries in the GT strengths, it is found that the asymmetry in the many-body wave functions is as important as the asymmetry in the single-particle wave functions. The many-body effects on the mirror asymmetry tend to be enhanced in the drip line nuclei.

Introduction

The properties of nuclei around the proton drip line are significant in understanding the rp-process nucleosynthesis and the related astrophysical phenomena (the novae and the X-ray bursts). It has been known that, around the β-stable line, the mirror nuclei show similar energy spectra as a result of the charge symmetry. Since the Coulomb force shifts the eigen-energies in an almost state-independent manner, the mirror symmetry holds to a large extent in terms of the excitation energies. Because it is not easy to access experimentally the nuclei near the proton drip line, their energy levels have often been assumed to be identical to those of their mirror counterparts in the astrophysical applications.

On the other hand, there is a class of levels which have sizable energy displacement in the excitation spectra between mirror nuclei, i.e. Thomas-Ehrman shift (TES); such as $1/2^+$ states of ^{17}O and ^{17}F. The TES may significantly influence the scenario of the rp-process nucleosynthesis. The TES, as well as other physical quantities with respect to the mirror asymmetry (except for the Nolen-Schiffer anomaly), is essentially the issue of the Coulomb force. As the lowest-order Coulomb effect, the proton-neutron difference in the single-particle (s.p.) wave functions has been considered to be the main source of the mirror asymmetry. However, some recent data on nuclei near the proton drip line have disclosed many-body effects on the mirror asymmetry. In Ref. 1, we pointed out the role of residual nuclear interaction (RNI) in the TES around ^{16}O.

In this research, we extensively study the many-body effects on mirror asymmetry in light sd-shell nuclei. In addition to the TES (i.e. the mirror asymmetry in the energy levels), the mirror asymmetry in the Gamow-Teller (GT) transition strengths will be investigated in the shell model framework.

TES and RNI reduction

The Thomas-Ehrman shift (TES) has been accounted for by the Coulomb energy reduction for loosely bound proton s orbits. In the proton-rich nuclei, the radial wave function of the proton orbit distributes over a range broader than that of the corresponding neutron orbit of its mirror counterpart. Because of the presence (or absence) of a centrifugal barrier, this effect is more significant for the lower ℓ orbit. Thus, in the light sd-shell region, the $(1s_{1/2})_p$ wave function has a long tail compared with the $(0d_{5/2})_p$ wave function, irrespective to the energy sequence of these orbits. As the last proton distributes more broadly, it gains less Coulomb repulsion from the other protons, which stay inside the core. This leads to a state dependence of the Coulomb energy, that is, it depends on the orbit that the last proton occupies. This is the lowest-order Coulomb effect, and has been considered as the main source of the TES.

The invention of the secondary-beam technique enables us to produce nuclei around the drip lines, particularly in the light mass region. As the nuclei around the proton drip line are investigated, it is disclosed that some mirror nuclei show large TES of about 1 MeV. In the ^{16}N–^{16}F mirror pair, even the ground-state spins do not match.[2] In Ref. 1, we investigated the role of RNI in the TES. Since the wave function of $(1s_{1/2})_p$ distributes with a long tail, its spatial overlap with another nucleon is depressed compared with that of $(1s_{1/2})_n$. Therefore nuclear force is expected to produce appreciably smaller matrix elements, as a secondary effect of the Coulomb force.

For the sake of simplicity, we analyzed the energy levels of ^{16}N–^{16}F in the shell model framework, assuming a model space consisting of only $0p_{1/2}$ holes and $(0d_{5/2}1s_{1/2})$ particles on top of the ^{16}O core.[1] If the core energy and the s.p. energies are determined from the ^{16}O, ^{17}O–^{17}F, and ^{15}N–^{15}O data, the relevant RNI matrix elements can be derived from the measured energies of the ^{16}N–^{16}F levels. Then the amount of RNI matrix elements turns out to be reduced in the proton-rich nuclei, by about 30% for those involving the $(1s_{1/2})_p$ orbit, and by 5% for the remaining elements. The conventional pn difference of the s.p. energies gives about 60% of the TES of this mirror pair, and the reduction in the amount of RNI matrix elements contributes by 40%, despite a

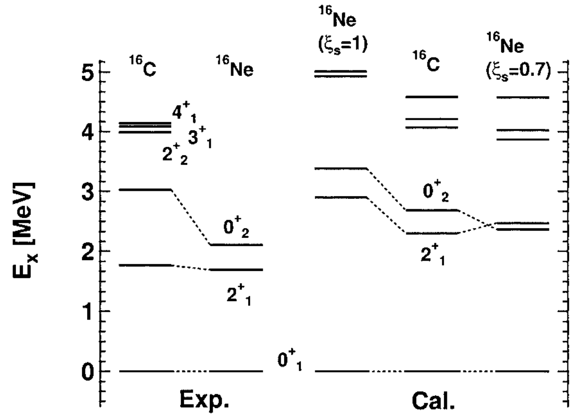

Fig. 1. Experimental and calculated (with $\xi_s = 1$ and 0.7) energy spectra of the ^{16}C–^{16}Ne mirror nuclei. Taken from Ref. 1.

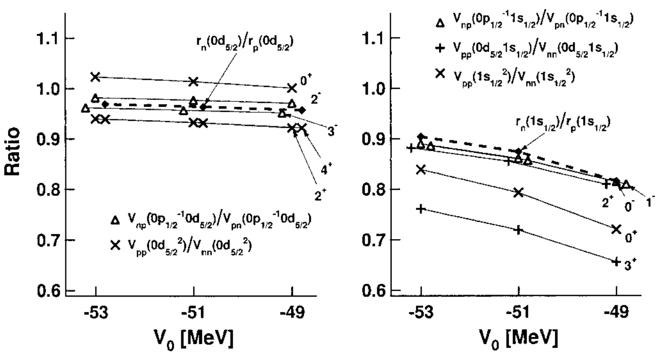

Fig. 2. Proton-neutron ratios of the RNI diagonal matrix elements in the WS+M3Y model, for the WS potential depth varied around $V_0 = -51$ MeV. The ratios involving the $1s_{1/2}$ orbit are shown in the right panel, and those without $1s_{1/2}$ but with $0d_{5/2}$ are in the left panel. The J values of the two-body states are indicated in the graph. The corresponding ratios with different V_0 values are connected by thin lines. The pn ratios of the rms radii of the s.p. orbits are also shown for $j = 0d_{5/2}$ (left panel) and $1s_{1/2}$ (right panel), by filled diamonds linked by thick dashed lines. Taken from Ref. 1.

higher-order effect of the Coulomb force. This suggests that many-body effects are important to the mirror asymmetry concerning the nuclei around the proton drip line. The many-body effects are comparable to the lowest-order s.p. effect, even though this analysis may be too simple for quantitative argument.

In a recent experiment, $E_x(0_2^+)$ of ^{16}Ne was determined to be lower than that of ^{16}C.[3] Although this is not reproduced when only the s.p. mechanism is considered for the TES of this mirror pair, we obtain the correct trend by taking into account the RNI reduction, as depicted in Fig. 1. In this calculation, the model space is assumed to be $(0p_{1/2})^{-2} \otimes (0d_{5/2}1s_{1/2})^2$. The pp or nn interaction in the sd-shell is taken from the USD interaction,[4] with the reduction factor ξ_s for the matrix elements involving $(1s_{1/2})_p$. While the TES of 0_2^+ is not reproduced for $\xi_s = 1$, i.e. the original USD elements of V_{pp}, the correct trend is obtained for $\xi_s = 0.7$. As was discussed in Ref. 1, nucleus dependence of the s.p. energies does not account for this tendency of the TES of 0_2^+.

Whereas we have shown phenomenological evidence of the RNI reduction, it is desired to confirm it from a microscopic standpoint. For this purpose, we computed the M3Y interaction[5] matrix elements using the s.p. wave functions in the Woods-Saxon potential.[6] The potential depth is slightly varied. In Fig. 2, the pn ratios of the matrix elements are illustrated for those involving $(1s_{1/2})$ (left panel) and for the others (right panel). The pn ratios of the rms radius of the $(0d_{5/2})$ orbit and that of the $(1s_{1/2})$ orbit are also presented. We view that the ratios of the RNI matrix elements are well correlated to the radial distribution of the relevant orbits, and that the RNI reduction can be as much as a few tens percent, in agreement with the above phenomenological argument for ^{16}N–^{16}F and ^{16}C–^{16}Ne.

Shell model calculation for light sd-shell nuclei

The proton-rich light sd-shell nuclei are located at the entrance to the rp-process nucleosynthesis. Since the (p, γ) reaction cross sections are highly affected by the resonance state of the daughter nucleus, the TES in the proton resonance region can be crucial to the scenario of the rp-process. The typical temperature in the rp-process is $T = 1.5 \times 10^9\,\mathrm{K} \cong 0.15\,\mathrm{MeV}$.[7] This indicates that, in order to predict the (p, γ) rate within an order of magnitude, we need the energy of the resonance state to be predicted at a 0.3 MeV accuracy. Despite recent progress, it is still difficult to measure all resonance levels relevant to the rp-process. From this viewpoint, prediction of the energy levels in the proton resonance regime to a 0.3 MeV accuracy or better will be an important task of the nuclear structure theory.

The mirror counterparts of the nuclei near the proton drip line are well inside the neutron drip line. As we have numerous data on the neutron-rich side and make reference to them, it is crucial to predict the TES to the desired accuracy. For the sd-shell nuclei, the shell model calculations using the so-called USD Hamiltonian[4] have been successfully applied. However, the exact isospin symmetry was assumed in the USD effective Hamiltonian. Because the observed TES in the light sd-shell region sometimes amounts to about 1 MeV, it is clearly required to modify the Hamiltonian for the astrophysical purposes. Hinted by the discussion in the preceding section, we take into account the RNI reduction for the $(1s_{1/2})_p$ orbit, as well as the pn difference of the s.p. energies.

In the first step, we restrict ourselves to the $18 \leq A \lesssim 20$ nuclei. For quantitative investigation, we adopt the full sd-shell, instead of the one without $0d_{3/2}$ in the preceding section. The s.p. energies are fixed using the ^{17}O and ^{17}F data. The RNI is determined from the USD with modifications; the most important one is the reduction in the amount of the matrix elements involving $(1s_{1/2})_p$, by a factor of 0.85. There are a few minor modifications carried out to fit the energy levels to those in the ^{18}O and ^{18}F data. The two-body Coulomb force is added to V_{pp}, based on the estimate using the WS s.p. wave functions.

By this Hamiltonian, we reproduce the energy levels of ^{18}O–^{18}Ne up to the TES, as shown in Fig. 3. Note that the reduction factor of 0.85 concerning $(1s_{1/2})_p$ is the single adjustable parameter to the TES for ^{18}O–^{18}Ne, after fitting the ^{18}O lev-

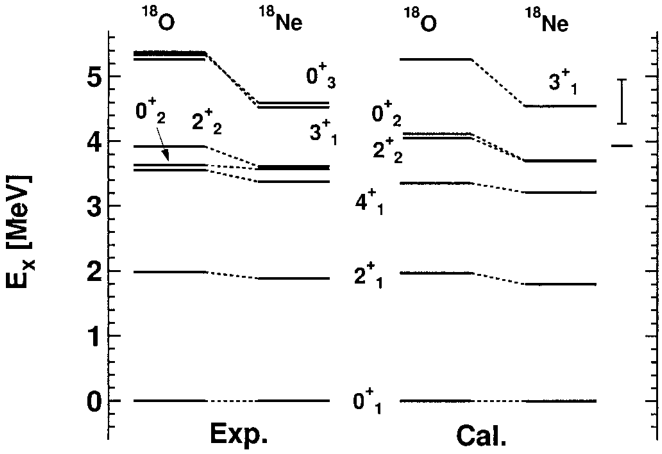

Fig. 3. Experimental and calculated energy levels of the ^{18}O–^{18}Ne mirror nuclei. The short horizontal bar and the error bar at the rightmost portion indicates S_p for ^{18}Ne and the Gamow window for $^{17}\mathrm{F}(p,\gamma)$, respectively.

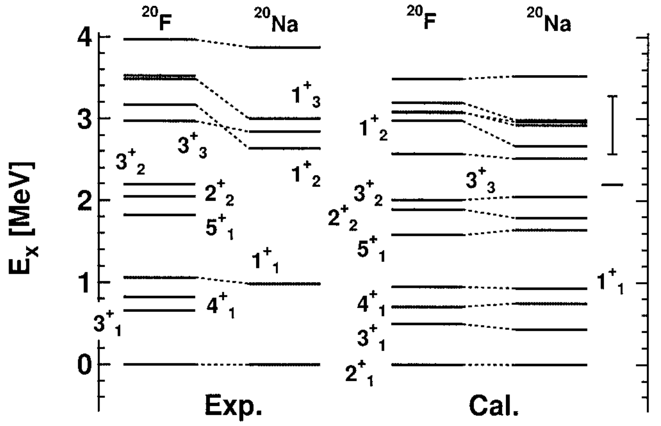

Fig. 4. Experimental and calculated energy levels of the ^{20}F–^{20}Na mirror nuclei. The short horizontal bar and the error bar at the rightmost portion indicates S_p for ^{20}Na and the Gamow window for $^{19}\mathrm{Ne}(p,\gamma)$, respectively.

els. The TES of several levels are reproduced using this single factor. In order to clarify the energy region of astrophysical importance, the proton separation energy (S_p) for ^{18}Ne and the Gamow window for $^{17}\mathrm{F}(p,\gamma)$ at $T = 1.5 \times 10^9\,\mathrm{K}$ are also shown in the figure. The recently confirmed 3^+_1 state of ^{18}Ne,[8)] whose relevance to the *rp*-process has been pointed out,[9)] is obtained at the correct energy. It is noted that the large TES places this level into the Gamow window for ^{18}Ne.

The energy levels of ^{20}F–^{20}Na are depicted in Fig. 4. For ^{20}F, the calculated levels are in good agreement with the experimental ones, although slightly compressed. We view that, while data are not numerous in the case of ^{20}Na, the observed TES are reproduced fairly well.

Since the captured proton in the *rp*-process has low kinetic energy, it is reasonably assumed to be in the *s*-wave. Therefore, the resonance states having the $(1s_{1/2})_p$ component predominantly contribute to the (p,γ) cross sections. Thus the levels relevant to the *rp*-process are basically picked up by the spectroscopic factors (C^2S) with respect to the $(1s_{1/2})_p$ orbit, as well as by their energies, whether or not the levels are within the Gamow window. We here list the levels of the $18 \leq A \leq 20$ nuclei which may play a key role in the *rp*-process, according to our shell model calculations:

$$
\begin{array}{llll}
^{18}\mathrm{Ne} & 3^+ & (4.6\,\mathrm{MeV})\,; & C^2S = 0.98\,, \\
^{19}\mathrm{Ne} & 3/2^+ & (6.8\,\mathrm{MeV})\,; & C^2S = 0.41\,, \\
 & 1/2^+ & (6.9\,\mathrm{MeV})\,; & C^2S = 0.14\,, \\
^{20}\mathrm{Na} & 1^+ & (2.7\,\mathrm{MeV})\,; & C^2S = 0.36\,, \\
 & 0^+ & (3.0\,\mathrm{MeV})\,; & C^2S = 0.58\,.
\end{array}
$$

Mirror asymmetry in GT strengths

We have so far focused on the TES, i.e. mirror asymmetry in energy levels. Let us turn to mirror asymmetry in other physical quantities. In general, there could be asymmetry in wave functions between mirror nuclei. For instance, the present isospin-breaking Hamiltonian also gives mirror asymmetry in the nuclear wave functions. It is of interest whether and how this wave function asymmetry is detected in the physical observables.

In the shell model picture, the mirror asymmetry in the wave functions can be separated into two parts: (i) the asymmetry in the s.p. wave functions, and (ii) the asymmetry in the many-body wave functions. The latter implies that in the energy eigenstates the amplitudes of the individual shell model configurations are not symmetric between the mirror nuclei. As in the case of the TES, the asymmetry in the s.p. wave functions is the lowest-order Coulomb effect, and has been discussed in previous researches. On the other hand, as a higher-order effect, the asymmetry in the many-body part has often been considered negligible.

There are few data on the mirror asymmetry in wave functions. Among various probes, the β-decay strengths are particularly interesting. Because of their pure isovector nature, mirror asymmetry in the β-decay strengths, if there is, is a clear evidence of the mirror asymmetry in the wave functions. In a recent experiment, a sizable mirror asymmetry in the GT strengths was observed between the $^{20}\mathrm{O} \to {}^{20}\mathrm{F}$ and $^{20}\mathrm{Mg} \to {}^{20}\mathrm{Na}$ decays.[10)] Together with the previously known mirror GT strengths,[11)] we discuss what this GT asymmetry indicates. Although the asymmetry in the decay to 1^+_2 was also discussed in Ref. 10, it is still unclear which 1^+ level of ^{20}Na should correspond to the 1^+_2 state of ^{20}F. In our calculation we obtain an additional 1^+ level at 2.7 MeV (see Fig. 4). For the decay from ^{20}O and ^{20}Mg, we focus only on the decay to 1^+_1, which is free of such problems.

We evaluate the ratios of the GT strengths between the mirror decays. Though it is not easy to fix the GT effective operator typically because of the quenching problem, the ratios are not subject to most of these issues. We should evaluate both the asymmetry ratios of the s.p. wave functions and of the many-body wave functions. The latter is given by the shell model calculations. In the shell model calculations, the s.p. wave functions are not handled explicitly; they are integrated out in advance. Since the GT operator has the $\sigma\tau$ form with no dependence on radial part, the s.p. ratio represents the overlaps between the neutron (proton) s.p. radial wave functions of the parent nucleus and the corresponding proton (neutron) s.p. wave functions of the daughter nucleus, on the

Table 1. GT ratios between mirror decays. See the text for the calculated results.

	Cal.(SP)	Cal.(MB)	Cal.(SP+MB)	Exp.
(A)	0.956	1.007	0.960	0.957
(B)	0.967	1.036	0.991	0.959
(C)	1.006	0.858	0.865	0.813

neutron-rich (proton-rich) side. Because of the centrifugal barrier, the overlaps for the d orbits are not quite different between the proton-rich and neutron-rich sides. However, the overlap on the proton-rich side should be smaller than on the neutron-rich side for the $1s_{1/2}$ orbit, because $(1s_{1/2})_p$ has a broad distribution. Although this overlap may depend on nuclides, we fix its amplitude to be 0.9. This approximation will be sufficient for the present purpose; in this paper we shall discuss whether or not the many-body effects are important on the GT asymmetry, rather than pursuing high accuracy.

We here consider the following three GT ratios:

$$\begin{array}{l}(\mathrm{A})\ B(\mathrm{GT};\ {}^{18}\mathrm{Ne}\,0^+_1 \to {}^{18}\mathrm{F}\,1^+_1)/B(\mathrm{GT};\ {}^{18}\mathrm{O}\,0^+_1 \to {}^{18}\mathrm{F}\,1^+_1),\\ (\mathrm{B})\ B(\mathrm{GT};\ {}^{20}\mathrm{Na}\,2^+_1 \to {}^{20}\mathrm{Ne}\,2^+_1)/B(\mathrm{GT};\ {}^{20}\mathrm{F}\,2^+_1 \to {}^{20}\mathrm{Ne}\,2^+_1),\\ (\mathrm{C})\ B(\mathrm{GT};\ {}^{20}\mathrm{Mg}\,0^+_1 \to {}^{20}\mathrm{Na}\,1^+_1)/B(\mathrm{GT};\ {}^{20}\mathrm{O}\,0^+_1 \to {}^{20}\mathrm{F}\,1^+_1).\end{array}$$

In all of the three cases, the ratio becomes unity if the mirror symmetry is maintained, and hence the deviation from unity represents the mirror asymmetry. In the denominator of (A), the initial and final states are reversed from the actual decay, and the GT strength is converted accordingly. The calculated GT strength ratios are shown in Table 1, in comparison with the measured ones. The second column (labeled by "SP") shows the ratios when only the asymmetry in the s.p. wave functions is taken into account, for which the many-body wave functions of the $Z > N$ nuclei are taken from their mirror counterparts, with the exchange of protons and neutrons. The third column (labeled by "MB") indicates the ratios evaluated only from the asymmetry in the many-body wave functions, using the many-body wave functions obtained from the present isospin-breaking Hamiltonian, and assuming the s.p. ratios to be unity. The fourth column presents the GT ratios with both of the wave function asymmetries.

While the asymmetry is dominated by the s.p. wave functions in the mirror decays (A) and (B), the s.p. mechanism does not account for the asymmetry in the mirror decay of (C). This is because the contribution of the $1s_{1/2}$ orbit is negligibly small in this decay (C). However, in the mirror decay (C) the asymmetry in the many-body wave functions is significant, turning the GT ratio to the correct direction. Thus, the GT asymmetry in the case (C) is concluded to be evidence of the asymmetry in the many-body wave functions, as a higher-order effect of the Coulomb force. It is noted that the extent of the asymmetry in this decay is even larger than that of the other ones. This indicates that the many-body effect is important also in the GT asymmetries despite a higher-order effect, as well as the lowest-order s.p. mechanism.

Combining both of the wave function asymmetries, we reproduce the GT ratios (A) and (C) quite well; 70% of the asymmetry in (C) is accounted for by the present calculation. However, the calculated ratio of (B) is too close to unity, compared with the experimental ratio, owing to the destructive contribution of the asymmetries in the s.p. and many-body wave functions. To improve this ratio, it will be necessary to increase the s.p. asymmetry or to decrease the asymmetry in the many-body wave functions. From microscopic viewpoints, it is unlikely for the s.p. wave function overlap ratio to be smaller than 0.9. Together with the remaining discrepancy in (C), this suggests that the present many-body wave functions may not be sufficiently accurate for fully quantitative discussion of the GT asymmetry, although the mirror asymmetry in these wave functions have correct tendency.

In the present approach, the many-body effects on the mirror asymmetry tend to be stronger as $|T_z|$ increases, since the difference between V_{pp} and V_{nn} is enhanced. Therefore the many-body effects on the asymmetry are expected to be prominent for the drip line nuclei. This is the reason why the many-body effects are enhanced in (C), and seems consistent with the recent experimental data.

The mirror asymmetry in the GT strengths were previously investigated for the p-shell nuclei.[12–14] Towner considered the asymmetry in the many-body wave functions (i.e. configuration mixing) due to the Coulomb force,[13] concluding that the many-body effects are not important in the transitions with $\log ft \lesssim 5$. This result is inconsistent with the ^{20}O–^{20}Mg data, obtained because only the Coulomb force is considered as the source of the many-body effects. Barker took into account the difference between V_{pp} and V_{nn},[14] according to the Ormand-Brown isospin-breaking Hamiltonian.[15] As in the present study, he showed that the many-body effects can be dominant for some GT strengths. In the present study, the loosely bound s-orbit effects (i.e. the reduction in the amount of the residual interaction) have been considered, which enhance the mirror asymmetry in the many-body wave functions.

Summary

We have discussed the mirror asymmetry in light sd-shell nuclei, mainly focusing on the many-body effects. In particular, the asymmetries in the energy levels and GT strengths, which reflect the asymmetry in the wave functions, are investigated. Although they are higher-order effects of the Coulomb force, the many-body effects are found to be as important as the lowest-order single-particle effects. There are even some cases in which the many-body effects dominate over the lowest-order single-particle effects, particularly in the drip line nuclei. It is emphasized that these many-body effects are manifested only recently, when the data on the nuclei near the proton drip line became available.

In addition to the conventional single-particle Coulomb reduction, the reduction in the amount of residual interaction matrix elements concerning the $(1s_{1/2})_p$ orbit plays a significant role in Thomas-Ehrman shifts. If both of these effects are taken into account, the energy levels of the $18 \leq A \lesssim 20$ nuclei, including the TES, are reproduced to quite a high accuracy by a shell model calculation. Based on this shell model calculation, several key states to the rp-process have been pointed out.

The GT strengths disclosed the mirror asymmetry in the nuclear wave functions. Analyzing the asymmetries in the GT strengths, we have found that the asymmetry in the many-body wave functions is as important as the asymmetry in the single-particle wave functions. By taking into account both asymmetries of the wave functions, the observed asymmetries in the GT strengths in the $18 \leq A \lesssim 20$ region are reproduced reasonably well. This seems to further confirm the validity of the present isospin-breaking shell model Hamiltonian.

These results convince us further that the present shell model approach using the Hamiltonian taking into account the reduction of the RNI is a promising tool for studying the mirror asymmetry. Although we have considered only the $18 \leq A \lesssim 20$ region, it will be desirable to investigate heavier nuclei. In regard to the *rp*-process, it is important to evaluate the Thomas-Ehrman shifts to an accuracy higher than 0.3 MeV. If the systematics of the single-particle energy shifts and of the reduction in the amount of the residual interaction are fixed, it may be possible to predict the TES to this accuracy. It is also interesting to investigate how the mirror asymmetry in wave functions affects physical quantities other than the GT transitions, *e.g.* the electromagnetic properties.

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